Mode Transitions in Low-Temperature Aerospace Plasmas

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Early Career Lecture

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Video
Plasma Mode Transitions

mode transition
	noun

a sudden, major transformation of the structure and/or dynamics of a plasma in response to a minor change to its operating conditions
Plasma Mode Transitions

Plasma Processing

Chabert et al., Plasma Sources Sci. Tech. 30.2 (2021): 024001

Fusion Energy


Plasma Mode Transitions

Hall Thrusters

Hollow Cathodes


Mode Transitions in Aerospace Plasmas

**Topic 1:** Mode transitions in helicon thrusters

**Topic 2:** Critical velocities for plasma aerocapture

**Topic 3:** Data-driven discovery of mode transition physics
Topic 1: Mode transitions in helicon thrusters

Topic 2: Critical velocities for plasma aerocapture

Topic 3: Data-driven discovery of mode transition physics
Goal: Understand plasma expansion and detachment from magnetic nozzle


Mode Transitions

Figure 3.10: Photographs (f/7.1, 1/30 s exposure) of the plasma source (top) operating in the capacitive (middle-top), inductive (middle-bottom), and helicon wave (bottom) modes.

![Photographs of plasma source in different modes](image_url)

![Graph depicting relationship between magnet current and density](image_url)

- Magnet Current, $I_B(A)$
- Density, $n_i(m^{-3})$
Low-Field Mode Transition (LFMT)


Low-Field Mode Transition (LFMT)

How does the mode transition field strength scale with thruster parameters?
LMFT Measurements

(a) $L_{bp} = 8.5$ cm

(b) $L_{bp} = 11.5$ cm

(c) $L_{bp} = 15.0$ cm

(d) $L_{bp} = 18.5$ cm

$B_0^\ast (G)$

Power, $P$ (W)

$L_{bp} = 8.5$ cm
- ▼ 2.5 mg/s
- □ 2.0 mg/s
- ◆ 1.5 mg/s
- ● 1.0 mg/s
- ■ 0.5 mg/s

$L_{bp} = 11.5$ cm
- ▼ 2.5 mg/s
- □ 2.0 mg/s
- ◆ 1.5 mg/s
- ● 1.0 mg/s
- ■ 0.5 mg/s

$L_{bp} = 15.0$ cm
- ▼ 2.5 mg/s
- □ 2.0 mg/s
- ◆ 1.5 mg/s
- ● 1.0 mg/s
- ■ 0.5 mg/s

$L_{bp} = 18.5$ cm
- ▼ 2.5 mg/s
- □ 2.0 mg/s
- ◆ 1.5 mg/s
- ● 1.0 mg/s
- ■ 0.5 mg/s
LMFT Measurements

How does the mode transition field strength scale for a converging B-field and m=0 spiral antenna?

More specifically...
Theoretical Model of the LMFT

Theoretical Model of the LMFT

RF Absorption

Thruster Global Model

Theoretical Model of the LMFT

\[ k k_z = \frac{\omega \omega_{pe}^2}{c^2 \omega_{ce}} \left( 1 - \frac{i \nu_{eff} k}{\omega_{ce} k_z} \right) \]

\[ \nu_{eff} = \nu_{ei} + \nu_{w} \]

Theoretical Model of the LMFT

![Graph showing the relationship between $P$ (W), $n_e$ ($m^{-3}$), $P_{\text{loss}}$, and $P_{\text{abs}}$ with $B = 10$ G]
Theoretical Model of the LMFT

\[ B = 30 \text{ G} \]
Theoretical Model of the LMFT

$P_{\text{abs}}$ vs. $n_e \,(m^{-3})$

- $P_{\text{loss}}$
- $B = 50 \, G$

W-E Mode Transition
Theoretical Model of the LMFT

- $P_{\text{loss}}$
- $P_{\text{abs}}$

$W$-E Mode Transition

$B = 50$ G

wave phase velocity $\gg$ electron thermal velocity
Theoretical Model of the LMFT

Wave absorption maximum at:

Using above plot:

\[ B_m = b_0 r_p n_e T_{eV}^{1/2} \]

\[ B^* \approx \chi B_m \]
Theoretical Model of the LMFT

Theoretical Model of the LMFT

Power balance:

\[ P_{\text{loss}} = n_e u_i \pi r_p^2 \left( \epsilon_{\text{ke}} + \epsilon_w + \epsilon_{\text{ion}}' \right) \]

Mass balance:

\[ B^* = \frac{\chi b_1 n_e P_{\text{RF}}}{2 r_p \mu(\gamma) e T_{\text{ev}} \sqrt{\gamma m_i}} \]

\[ T_{\text{ev}} \approx \left( \frac{b_0 c_j u_n r_p^2 \sqrt{m_i/\gamma}}{b_1 mL} \right)^{1/n} \]

\[ B^* = C_1 \frac{P_{\text{RF}}}{r_p \sqrt{m_i} \left( \frac{mL}{c_j r_p^2} \right)^{1/n}} \]
LMFT Scaling with Thruster Parameters

(a) $L_{bp} = 18.5 \text{ cm}$

(b) $L_{bp} = 15.0 \text{ cm}$

(c) $L_{bp} = 11.5 \text{ cm}$

$P_{RF}$ vs. $B_0^*$

$P_{RF}(mL)^{1/n}$ vs. $B_0^*$

$R_m P_{RF}(mL)^{1/n}$ vs. $B_0^*$
LMFT Scaling with Thruster Parameters

Transition occurs when Landau damping of wave energy can longer sustain plasma.
Mode Transitions in Aerospace Plasmas

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Generating Forces using Magnetic Fields in Space

Magnetic Sail


Plasma Magnetic Sail (M2P2)


Plasma Magnetoshell


Plasma Magnetoshell Aerocapture

“Plasma parachute” enables high-velocity aerocapture at lower densities compared to aeroshells

Theoretical modeling and mission analysis:
- Neptune: enables 1,000 kg orbiter spacecraft
- Mars: enables orbit insertion of 60 MT payload
- Significant reduction in TPS requirements

Proof-of-concept tests using pulsed MPDT exhaust plume
- Thrust stand $\rightarrow$ 1,000X increase in drag over aerodynamic

Magnetoshell Analytical Model

Original Model (Kirtley, 2012):
- Magnetic “layers”
- Cylindrical plasma
- No particle trajectories
- \textit{Ad hoc} drag-area
- Partial mass/energy transfer

Our Model (Kelly and Little, 2019):
- Dipole magnetic field
- Plasma dipole equilibrium
- Particle trajectories
- Self-consistent drag area
- Full mass/energy transfer

How well does the magnetoshell utilize mass/energy from the flow?
Behavior of Newly Ionized Flow Particles

\[ \frac{d\hat{u}}{dt} = \frac{1}{\rho_L} \hat{u} \times \hat{B} \]

\[ \frac{d\hat{r}}{dt} = \hat{u} \]

\[ \rho_L = \frac{m_i u_{\infty}}{q_i B_0 r_c} \]

\[ \hat{\psi}^* = \sqrt{2\rho_L} \]
“Stream” neutral continuity:

\[
\nabla \cdot (n_{sn} \mathbf{u}_{sn}) = -R_{tot} n_{sn} n_e \\
R_{tot} = R_{ion} + R_{cx} \\
\hat{n}_{sn} = \exp(-\zeta_{tot} I_n) \\
I_n = \int \frac{\psi}{\psi_r} \, d\zeta \\
\zeta_{tot} \equiv \frac{R_{tot} n_{e,r}}{u_\infty / r_c}
\]

(ionization + charge exchange)
(magnetic topology + \(n_e\)-distribution)

(Ionization frequency)
(Neutral transit frequency)

\[
\dot{N}_{cap} = \oint_{\psi > \psi^*} R_{tot} n_{sn} n_e \, d\mathbf{V} \\
P_{cap} = \frac{1}{2} m_{sn} u_\infty^2 \dot{N}_{cap} \\
\dot{N}_{cap} = \hat{P}_{cap} = \zeta_{tot} I_{sn} (\rho_L, \zeta_{tot})
\]

(Characteristic Larmor radius)
(Magnetic coil radius)

Flow Utilization and Mode Transition

How well does the magnetoshell utilize mass/energy from the flow?

Power (and mass) balance in $\psi^*$:

$$\hat{P}_{\text{in}} + \hat{P}_{\text{inj}} = \frac{\hat{N}_i}{\hat{\tau}_e} (\hat{T}_e + \hat{\varepsilon}_{\text{ion}}) + \hat{N}_{\text{cap}} \hat{T}_i$$

- $\hat{P}_{\text{cap}} + \hat{P}_{\text{inj}}$ (Power captured from the stream neutrals)
- $\hat{P}_{\text{out}}$ (Power injected into dipole plasma)
- $\hat{N}_i (\hat{T}_e + \hat{\varepsilon}_{\text{ion}})$ (Diffusion of electron thermal power)
- $\hat{N}_{\text{cap}} \hat{T}_i$ (Net diffusion of ion + 2n thermal power)
- $\hat{P}_{\text{inj}}$ (Power injected into dipole plasma)

*Critical condition on electron confinement time

RF Absorption

Thruster Global Model

Flow Absorption

Magnetoshell Global Model

MODE TRANSITION

Increasing electron energy confinement time critical to performance!
Do our predictions hold for “standard” diffusion models?

Mode Transition with Increasing Flow Velocity

\[ \zeta_s^* = \frac{R_s n_e^*}{u_\infty/r_c} \]

(Interaction Strength)
How does the performance compare to the original model?
Neptune Orbiter: \( r_c = 2 \) meters
\( u_\infty = 25 \) km/s

Performance Comparison

**Flight envelope of magnetoshell viability at Neptune**

Do the benefits still outweigh the added mass/complexity?
Magnetoshell Aerocapture for Neptune Orbiter Mission

+70% payload mass
-30% aerocapture system mass
-30% peak heat flux, -45% total heat load
16x improvement in variable drag control

Variable drag increases entry corridor width

\[ \beta_1 = 5 \text{ kg/m}^2 \]
\[ \beta_1 = 10 \text{ kg/m}^2 \]
\[ \square \beta \text{ ratio} = 2 \]
\[ \triangle \beta \text{ ratio} = 5 \]
\[ \bigcirc \beta \text{ ratio} = 10 \]
\[ \blacktriangledown \beta \text{ ratio} = 20 \]

* Data show for Venus entry

Magnetoshell deploys and continuously modulates ballistic coefficient, $\beta(t)$

Spacecraft exits atmosphere on elliptic orbit

Spacecraft approaches target atmosphere on hyperbolic trajectory

Magnetoshell is jettisoned post-maneuver
Magnetoshell Aerocapture for Neptune Orbiter Mission

Variable drag increases entry corridor width

How do we test this concept?

Magnetoshell deploys and continuously modulates ballistic coefficient, $\beta(t)$

Spacecraft exits atmosphere on elliptic orbit

Spacecraft approaches target atmosphere on hyperbolic trajectory

Magnetoshell is jettisoned post-maneuver
Flow Requirements of Magnetoshell Experiments

Wind tunnels unable to produce relevant flow regime!
UW Plasma Magnetoshell Aerocapture Experiment

Diagram showing the setup of the experiment with labeled components such as RF power, gas injection, plasma, B-field, neutralizer plate, neutral beam, electromagnets, DC bias voltage, vacuum chamber, permanent magnet, and momentum flux pendulum.
UW Plasma Magnetoshell Aerocapture Experiment
UW Plasma Magnetoshell Aerocapture Experiment

RF power (W)

MFS force ($\mu$N)

$V_{\text{plate}}$ (V)

- 0
- -15
- -30
- -45

RF power (W)

400 600 800 1000 1200 1400

0 50 100 150 200
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Mode Transitions in Aerospace Plasmas

- RF Absorption
- Thruster Global Model
- Flow Absorption
- Magnetoshell Global Model
- Driving Physics
- Plasma Configuration

MODE TRANSITION

MICROSCOPIC PROCESSES

MACROSCOPIC ORGANIZATION

Mode Transitions in Aerospace Plasmas
Sparse Identification of Nonlinear Dynamics (SINDy)

I. True Lorenz System

\[
\begin{align*}
\dot{x} &= \sigma(y - x) \\
\dot{y} &= x(\rho - z) - y \\
\dot{z} &= xy - \beta z.
\end{align*}
\]

II. Sparse Regression to Solve for Active Terms in the Dynamics

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{z}
\end{bmatrix} =
\begin{bmatrix}
x & y & z & x^2 & xy & xz & y^2
\end{bmatrix}
\begin{bmatrix}
\xi_1 \\
\xi_2 \\
\xi_3
\end{bmatrix}
\]

III. Identified System

\[
\begin{align*}
\dot{x} &= \Theta(x^T)\xi_1 \\
\dot{y} &= \Theta(x^T)\xi_2 \\
\dot{z} &= \Theta(x^T)\xi_3
\end{align*}
\]

L-H Mode Transition in Magnetically-Confined Plasma


Dynamical Model of the L-H Transition

Turbulence envelope equations in 0D provide dynamical model for L-H transition

\[ \frac{\partial \mathcal{E}}{\partial t} = \mathcal{E} \mathcal{N} - a_1 \mathcal{E}^2 - a_2 V^2 \mathcal{E} - a_3 V_{ZF}^2 \mathcal{E} \]

\[ \frac{\partial \mathcal{N}}{\partial t} = -c_1 \mathcal{E} \mathcal{N} - c_2 \mathcal{N} + Q \]

\[ Q = 0.01 t \]
\[ a_1 = 0.2 \]
\[ a_2 = a_3 = 0.7 \]
\[ b_1 = 1.5 \]
\[ b_2 = b_3 = 1.0 \]
\[ c_1 = 1.0 \]
\[ c_2 = 0.5 \]
\[ d = 1.0 \]

Application of SINDy to L-H Mode Transition

Can SINDy find coefficients using LCO data?

\[
Q = 0.01t \\
a_1 = 0.2 \\
a_2 = a_3 = 0.7 \\
b_1 = 1.5 \quad b_2 = b_3 = 1.0 \quad c_1 = 1.0 \quad c_2 = 0.5 \quad d = 1.0
\]

\[
\partial_t N = -c_1 \mathcal{E} N - c_2 N + Q
\]
Application of SINDy to L-H Mode Transition

What if we add “unknown” physics?

\[
\partial_t V_{ZF} = b_1 \frac{\mathcal{E} V_{ZF}}{1 + b_2 V^2} - b_3 V_{ZF} + b_4 \mathcal{E}^2 V_{ZF}
\]

\[b_4 = 0.5\]
Application of SINDy to Mode Transitions

1. Diagnostic advances → more data, higher quality
2. Identification of “macro”— scale physical drivers
3. Characterization of link between “macro” and “micro” scale physics
Conclusions

- Plasmas exhibit bizarre, oftentimes unexpected behavior
- Understanding "non-ideal" physics critical for space technology
- Exciting new methods on the horizon enabled by “big data”
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QUESTIONS?