Modeling of Turbulent Radiative Shocks with Applications to High Energy Density Physics and Astrophysics

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Investigation models turbulent radiation hydrodynamics in the diffusion approximation and evaluates its effects on radiative blast waves.

- Radiation transport, shock physics, and turbulence are intimately coupled in several HEDP and astrophysical environments
  - Supernovae, competing processes in stellar life cycles, black hole dynamics
  - Z-pinches, high-energy laser experiments

- Blast waves created by such phenomena are susceptible to instabilities
  - Rayleigh-Taylor, Kelvin-Helmholtz, Richtmyer-Meshkov

- Theoretical and experimental studies typically focus on one or two of the processes: radiation, shock physics, turbulence

- This study models radiation hydrodynamics via equilibrium diffusion and turbulence by a Reynolds-averaged Navier-Stokes (RANS) model.
Radiation fields directly influence hydrodynamics in extreme temperature and pressure environments.

- Radiation to hydrodynamic pressure ratio $= a_R T^4 / (3 \rho c_s^2)$

- Radiation quickly dominates system as temperature increases
  
  $\frac{p_M}{p_R} (T = 10^6 K) \approx 5.95 \times 10^{-3}$,  
  $\frac{p_M}{p_R} (T = 10^7 K) \approx 6.55 \times 10^{-6}$
Reynolds and Favre decompositions along with gradient-diffusion approximations are used to provide closed turbulent transport equations.

- Reynolds and Favre averaging can be expressed as ordinary and density-weighted temporal means, respectively:
  \[ \bar{\phi} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t}^{t+\tau} \phi(x, t) \, dt \quad \text{and} \quad \bar{\rho} = \lim_{\tau \to \infty} \frac{1}{\tau} \int_{t}^{t+\tau} \rho(x, t) \varphi(x, t) \, dt \]

- Reynolds and Favre decompositions are given by:
  \[ \rho = \bar{\rho} + \rho' \quad \text{and} \quad p = \bar{p} + p' \quad \text{with} \quad E_R = \bar{E}_R + E'_R \quad \text{and} \quad F_j = \bar{F}_j + F'_j \]
  \[ v_j = \bar{v}_j + v''_j \quad \text{with} \quad U = \bar{U} + U'' \quad \text{and} \quad T = \bar{T} + T'' \]

- Averaging system of interest and using decompositions leads to fluctuating correlations closed via gradient-diffusion closures.

- Gradient-diffusion approximation uses turbulent kinetic energy, \( K \), and dissipation rate, \( \epsilon \), to form the turbulent viscosity needed for the closures:
  \[ \nu_t = C_\mu \frac{K^2}{\epsilon} = \frac{\mu_t}{\bar{\rho}} \]
Turbulent radiative gas dynamics is achieved by Reynolds averaging equilibrium diffusion model and generalizing gradient-diffusion closures. Given total energy \( \bar{\rho} \bar{\mathcal{E}} = \bar{\rho} \left( \bar{\nu}^2 / 2 + \bar{U} + K \right) + \bar{E}_R \), total pressure \( \bar{p}^* = \bar{p}_M + \bar{p}_R \), Reynolds stress tensor \( \tau_{ij} \), and radiative pressure dilatation \( \Pi^* \), the first five model equations are:

**Mass**
\[
\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{v}_j) = 0
\]

**Momentum**
\[
\frac{\partial}{\partial t} (\bar{\rho} \bar{v}_i) + \frac{\partial}{\partial x_j} (\bar{\rho} \bar{v}_i \bar{v}_j) = \bar{\rho} g_i - \frac{\partial \bar{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}
\]

**Total Energy**
\[
\frac{\partial}{\partial t} (\bar{\rho} \bar{\mathcal{E}}^*) + \frac{\partial}{\partial x_j} \left[ (\bar{\rho} \bar{\mathcal{E}}^* + \bar{p}^*) \bar{v}_j \right] = \bar{\rho} g_i \bar{v}_i - \frac{\partial}{\partial x_j} \left[ \left( \frac{\bar{p}^* + \bar{E}_R}{\bar{\sigma}_\rho} \right) \nu_t \frac{\partial \bar{\rho}}{\partial x_j} \right]
\]

\[
- \frac{\partial}{\partial x_j} \left( \tau_{ij} \bar{v}_i \right) + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\bar{\sigma}_U} \frac{\partial \bar{U}}{\partial x_j} + \frac{\mu_t}{\bar{\sigma}_K} \frac{\partial K}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \frac{4}{3} \frac{\nu_t}{\bar{\sigma}_{E_R}} \frac{\partial \bar{E}_R}{\partial x_j} \right) - \frac{\partial \bar{F}_j^N}{\partial x_j}
\]

**Turbulent K Energy**
\[
\frac{\partial}{\partial t} (\bar{\rho} K) + \frac{\partial}{\partial x_j} (\bar{\rho} K \bar{v}_j) = - \frac{\nu_t}{\bar{\sigma}_\rho} \frac{\partial \bar{\rho}}{\partial x_j} \frac{\partial \bar{p}^*}{\partial x_j} - \tau_{ij} \frac{\partial \bar{v}_i}{\partial x_j} - \bar{\rho} \epsilon + \Pi^* + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\bar{\sigma}_K} \frac{\partial K}{\partial x_j} \right)
\]

**Dissipation Rate**
\[
\frac{\partial}{\partial t} (\bar{\rho} \epsilon) + \frac{\partial}{\partial x_j} (\bar{\rho} \epsilon \bar{v}_j) = - \frac{\epsilon}{K} \left[ C_{\epsilon 0} \frac{\nu_t}{\bar{\sigma}_\rho} \frac{\partial \bar{\rho}}{\partial x_j} \frac{\partial \bar{p}^*}{\partial x_j} + C_{\epsilon 1} \tau_{ij} \frac{\partial \bar{v}_i}{\partial x_j} + C_{\epsilon 2} \bar{\rho} \epsilon - C_{\epsilon 3} \Pi^* \right]
\]

\[
+ \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\bar{\sigma}_\epsilon} \frac{\partial \epsilon}{\partial x_j} \right)
\]
Turbulence contributions introduced via mean radiative flux introduce need for transport equations for density and temperature variances.

- Classical and mean radiative fluxes with opacity model $\Sigma_A(\rho, T)$ for $1 \leq n \leq 3$ are:

$$ F_j^n = \frac{c}{3 \Sigma_A(\rho, T)} \frac{\partial E_R}{\partial x_j}, \quad \Sigma_A(\rho, T) = \beta \frac{\rho}{T^n} $$

$$ F_j^{n=1} = -\frac{a_R c T}{3 \beta \bar{\rho}} \left[ \left( 1 + 2 C_T \frac{\sqrt{\rho' r T^{\prime 2}/\bar{\rho}}}{\bar{\rho}} \right) \frac{\partial T^4}{\partial x_j} - \frac{2}{\lambda_T} \frac{\sqrt{2 \rho' r}}{\pi} \right] $$

$$ -\frac{a_R c}{3 \beta \bar{\rho}} \left[ \frac{\partial}{\partial x_j} \left( \bar{T}^5 - \bar{T} T^4 - C_T \frac{\sqrt{\rho' r T^{\prime 2}/\bar{\rho}}}{\bar{\rho}} \right) - \frac{2}{\lambda_T} \left( \bar{T}^5 - T^4 \bar{T} \right) + C_T \bar{T}^4 \frac{\partial}{\partial x_j} \left( \frac{\sqrt{\rho' r T^{\prime 2}/\bar{\rho}}}{\bar{\rho}} \right) \right] $$

- Compressible turbulent and PDF closures are used in density variance, $\rho'^2$, and temperature variance, $\bar{T}^\prime r$, transport equations development.

Density Variance:

$$ \frac{\partial \rho'^2}{\partial t} + \frac{\partial}{\partial x_j} (\rho'^2 \bar{v}_j) = \frac{2 \nu_t}{\sigma_p} \left( \frac{\partial \bar{\rho}}{\partial x_j} \right)^2 - \rho'^2 \frac{\partial \bar{v}_j}{\partial x_j} - C_{\rho^2} \frac{\epsilon}{K} \frac{\rho'^2}{\bar{\rho}} - \frac{2 \rho^2 \Pi^*}{\gamma \bar{\rho}} + \frac{\partial}{\partial x_j} \left( \frac{\nu_t}{\sigma_p^2} \frac{\partial \rho'^2}{\partial x_j} \right) $$

Temperature Variance:

$$ \frac{\partial}{\partial t} \left( \bar{\rho} \bar{T}^\prime r^2 \right) + \frac{\partial}{\partial x_j} \left( \bar{\rho} \bar{T}^\prime r^2 \bar{v}_j \right) = 2 \left[ \frac{\mu_t}{\sigma_U} \left( \frac{\partial \bar{T}}{\partial x_j} \right)^2 - (\gamma - 1) \bar{\rho} \bar{T}^\prime r^2 \frac{\partial \bar{v}_j}{\partial x_j} \right] $$

$$ -2 (\gamma - 1) C_T \frac{\epsilon}{K} \bar{T} \sqrt{\rho'^2 \bar{T}^\prime r^2} + \frac{\partial}{\partial x_j} \left( \frac{\mu_t}{\sigma_U} \frac{\partial \bar{T}^\prime r^2}{\partial x_j} \right) $$
Rankine-Hugoniot jump relations ensure total mass, momentum, and energy conservation and provide post shock relations

- Exact relations for profiles behind strong turbulent-radiative shocks as functions of shock speed, $\tilde{\nu}_s$, are given by

**Density:**

$$\frac{\bar{\rho}_2}{\bar{\rho}_1} = \frac{(\gamma + 1) \bar{p}^*_2 + (\gamma - 1) \tau_2 + 2(4 - 3 \gamma) \bar{p}_{R,2}}{(\gamma - 1) \bar{p}^*_2} \left[ 1 + \frac{2}{\bar{p}^*_2} \left( \frac{\tau_2}{2} - \bar{\rho}_1 K_2 - \frac{\Gamma^*_2}{\bar{\nu}_1} \right) \right]^{-1}$$

**Velocity:**

$$\tilde{\nu}_2 = \tilde{\nu}_s \left\{ 1 - \frac{(\gamma - 1) \bar{p}^*_2}{(\gamma + 1) \bar{p}^*_2 + (\gamma - 1) \tau_2 + 2(4 - 3 \gamma) \bar{p}_{R,2}} \left[ 1 + \frac{2}{\bar{p}^*_2} \left( \frac{\tau_2}{2} - \bar{\rho}_1 K_2 - \frac{\Gamma^*_2}{\bar{\nu}_s} \right) \right] \right\}$$

**Pressure:**

$$\bar{p}^*_2 = \bar{\rho}_1 \tilde{\nu}^2_s \left\{ 1 - \frac{(\gamma - 1) \bar{p}^*_2}{(\gamma + 1) \bar{p}^*_2 + (\gamma - 1) \tau_2 + 2(4 - 3 \gamma) \bar{p}_{R,2}} \left[ 1 + \frac{2}{\bar{p}^*_2} \left( \frac{\tau_2}{2} - \bar{\rho}_1 K_2 - \frac{\Gamma^*_2}{\bar{\nu}_s} \right) \right] \right\}$$

$$\Gamma^* = \frac{2}{\sigma_\rho} \frac{3 \bar{p}_R}{\bar{\rho}} \frac{\sigma_\rho}{\sigma_U} \frac{\partial \bar{\rho}}{\partial x_j} - \frac{\mu_t}{\sigma_U} \frac{\partial \tilde{U}}{\partial x_j} - \frac{\mu_t}{\sigma_K} \frac{\partial K}{\partial x_j} - \frac{4 \nu_t}{\sigma_{E_R}} \frac{\partial \bar{p}^*_2}{\partial x_j} + F_{n,F}$$

- As a check, results for a strong classical shock are obtained when removing turbulence and radiative effects

$$\frac{\rho_2}{\rho_1} = \frac{\gamma + 1}{\gamma - 1}, \quad \nu_2 = \frac{2 \nu_s}{\gamma + 1}, \quad p_2 = \frac{2 \rho_1 \nu_s^2}{\gamma + 1}$$
Weighted Essentially Non Oscillatory (WENO) and Riemann solvers will be used to simulate proposed turbulent radiation hydrodynamics model

- Sod reference problem is used to test early stage computational work
- Problem depicts two regions ($\gamma = 1.4$) under conditions:
  - $\rho_1 = 1.0$, $p_1 = 1.0$, $v_1 = 0 \parallel \rho_5 = 0.125$, $p_5 = 0.10$, $v_5 = 0$
Physics of underlying turbulent radiative shocks has been investigated
- An equilibrium diffusion model describes radiation hydrodynamics
- A four-equation Reynolds-averaged Navier-Stokes (RANS) model is used to describe turbulence effects
- Gradient-diffusion and similarity closures were generalized to account for radiative effects

WENO methods and approximate Riemann solvers will be used for conducting numerical investigations

Particular interest lies in studying these processes in planar, cylindrical, and spherical geometries for applications relevant to supernovae, black hole dynamics, high-energy laser experiments, and Z-pinches

Propose experiments that can be used to verify this model

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