Evaluation of RF Power Absorption and Electric and Magnetic Field Enhancements due to Surface Roughness

Peng Zhang, Y. Y. Lau, and R. M. Gilgenbach

Department of Nuclear Engineering and Radiological Sciences
University of Michigan
Ann Arbor, MI, USA, 48109-2104

Michigan Institute for Plasma Science and Engineering
1st Annual Graduate Student Symposium
September 29, 2010

This work was supported by AFOSR, L-3 Communications, and Northrop-Grumman Corporation.

Introduction

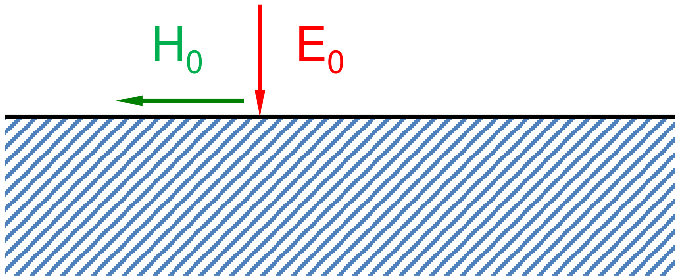
- Surface roughness may assume many forms
  - Impurities or foreign objects
  - Manufacture tolerance, same material as the surface
  - Grain boundaries

- Surface roughness may lead to
  - Enhanced RF power loss
  - Local electric field enhancement, breakdown
  - Local magnetic field enhancement, quenching in a superconducting cavity, i.e., rapid loss of superconductivity

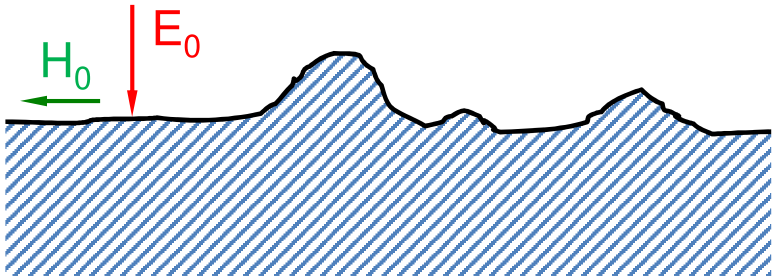
Outline

- Model
- RF absorption on flat metal surface
- RF absorption due to surface roughness
- Electric field enhancement due to surface roughness
- Magnetic field enhancement due to surface roughness
- Conclusion

Rough Surface



Pristine Surface



Rough Surface

(1)Cause enhanced heating

(2)Cause local field enhancement

E_{rf} ⟹ Field Emission ⟹ Breakdown

H_{rf} ⟹ Quenching of Superconductor Cavity

Approach

- Surface roughness is represented by a hemispherical protrusion of radius $a << \lambda$

  ε, μ, and σ of protrusion assume arbitrary values

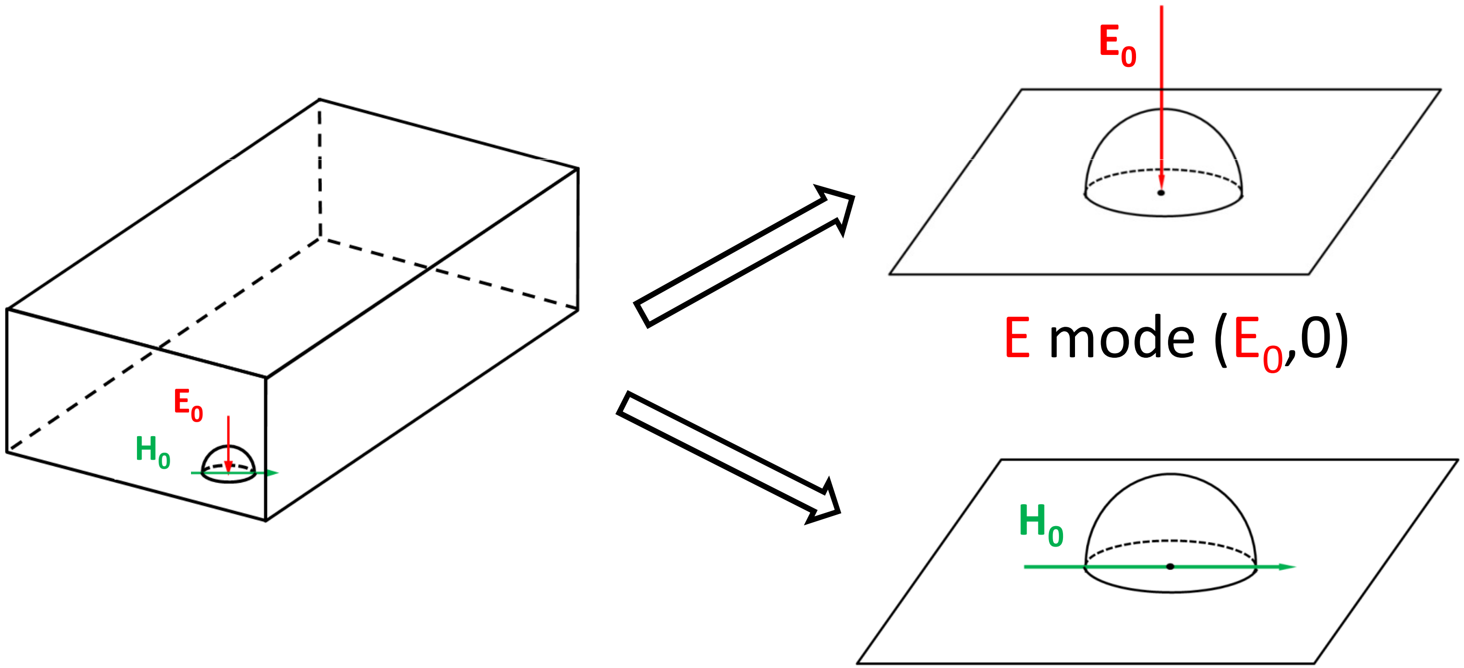
  - protrusion may represent foreign object or same material

- Accurately and self-consistently calculate RF electric field and RF magnetic field in presence of protrusion

- Perturbed eigenvalue gives enhanced RF loss, Perturbed eigenfunction gives RF field enhancements

Hemispherical protrusion on the surface ($a \ll \lambda_{exterior}$)

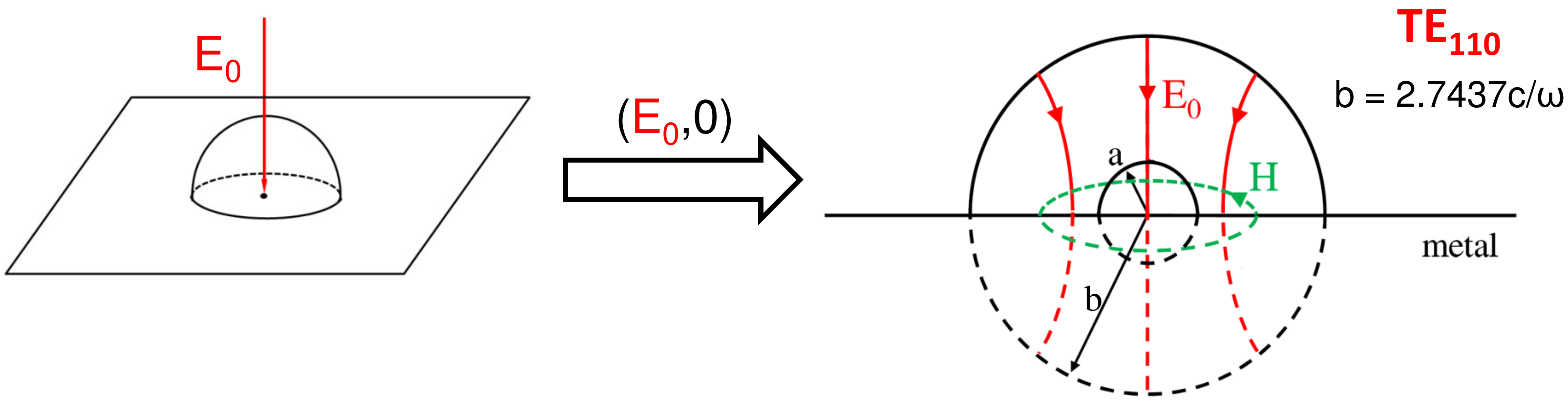
$$(E_0, H_0) = \underbrace{(E_0, 0)}_{\text{E mode}} + \underbrace{(0, H_0)}_{\text{M mode}}$$

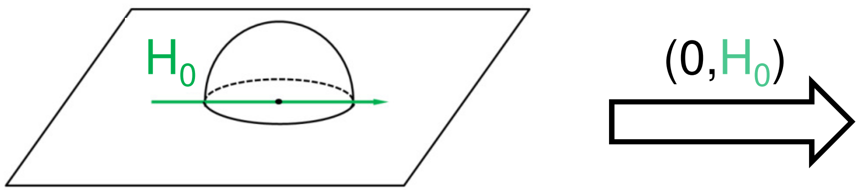


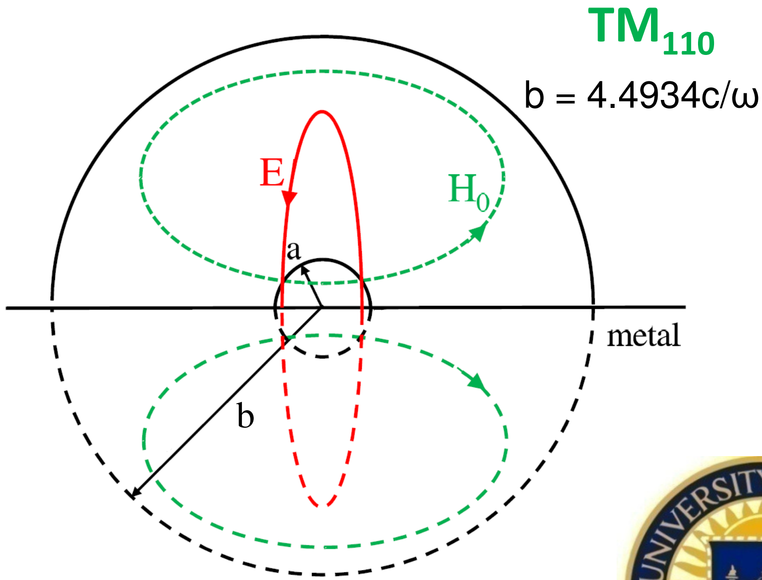
E mode $(E_0, 0)$

M mode $(0, H_0)$

Hemispherical protrusion on the surface







The irregular geometry "hemispherical protrusion on a surface" transformed into an equivalent, but highly symmetrical problem "spherical particulate in a spherical cavity"

Hemispherical protrusion on the surface

For both TE_{110} & TM_{110} mode,

(A) Perturbation on Eigenvalue gives power dissipated by particulate[1]

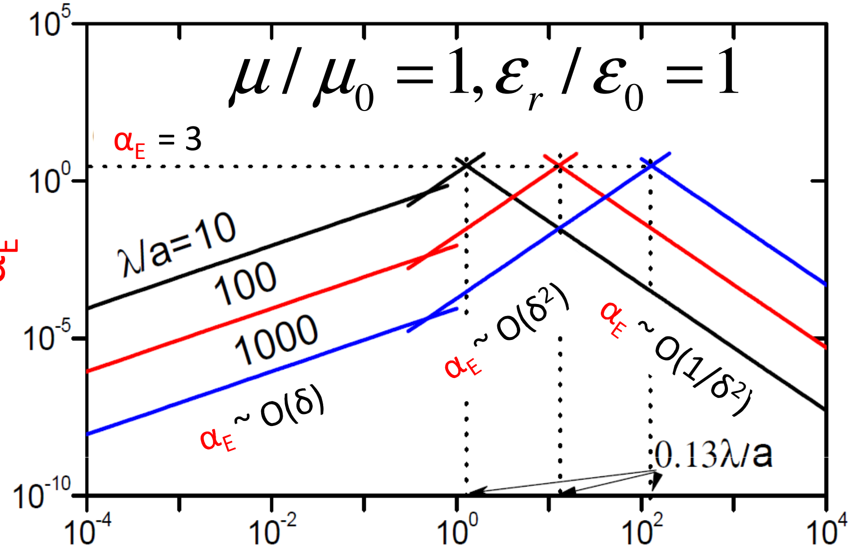
$$2\omega_i = \frac{P_d}{U}$$

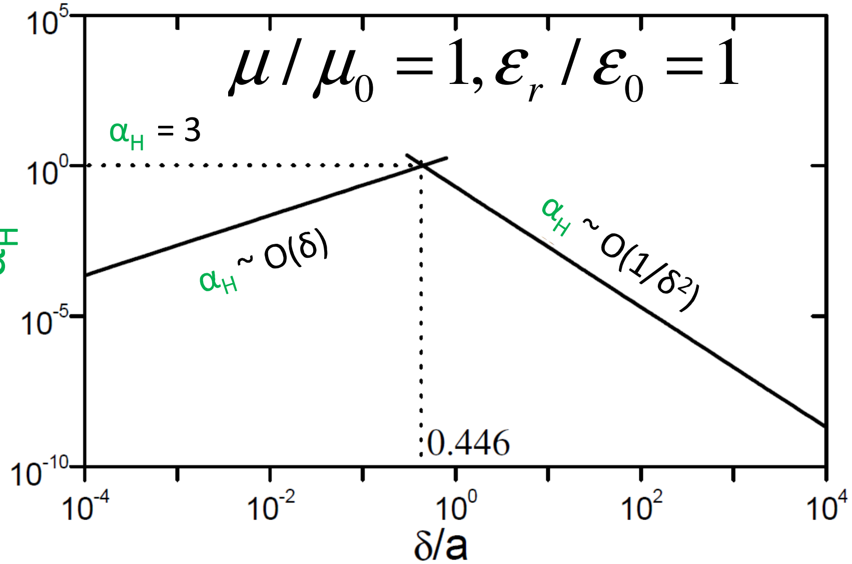
(B) Perturbation on Eigenfunction gives RF field enhancements

Note: (a) (ε_r, σ, μ) of protrusion may be arbitrary.
(b) Perturbed TE_{110} & TM_{110} mode calculated exactly, consistent with full set of Maxwell equations.

[1] W. Tang, H. Bosman, Y. Y. Lau and R. M. Gilgenbach, J. Appl. Phys. 97, 114915 (2005)

RF Power absorption due to small hemispherical protrusion on the surface





$$P_E = \alpha_E \omega \left(\frac{1}{2} \varepsilon_0 E_0^2 \right) V$$

α_E: Electrical polarizability

$$V = \frac{2\pi}{3} a^3 \text{ =Volume of protrusion}$$

$$P_M = \alpha_H \omega \left(\frac{1}{2} \mu_0 H_0^2 \right) V$$

α_H: Magnetic polarizability

δ = skin depth

Comparison of RF Power absorption due to uncorrelated small hemispherical protrusions

I. If protrusions & flat surface of **same** conducting materials, $\delta = \delta_s$

$$R = \frac{\sum P_{protrusion}}{P_{flat}} = 3f_{protrusion}$$

$$f_{protrusion} = A_{protrusion}/A_{flat}$$

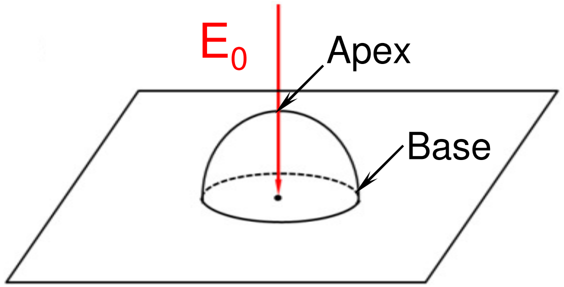
II. If protrusions are foreign objects with maximum α_E, the maximum ohmic loss through the RF electric field is

$$R = R_{max}(TE) = \left(\frac{4a}{\delta_s}\right) f_{protrusion}$$

III. If protrusions are foreign objects with maximum α_H, the maximum ohmic loss through the RF magnetic field is

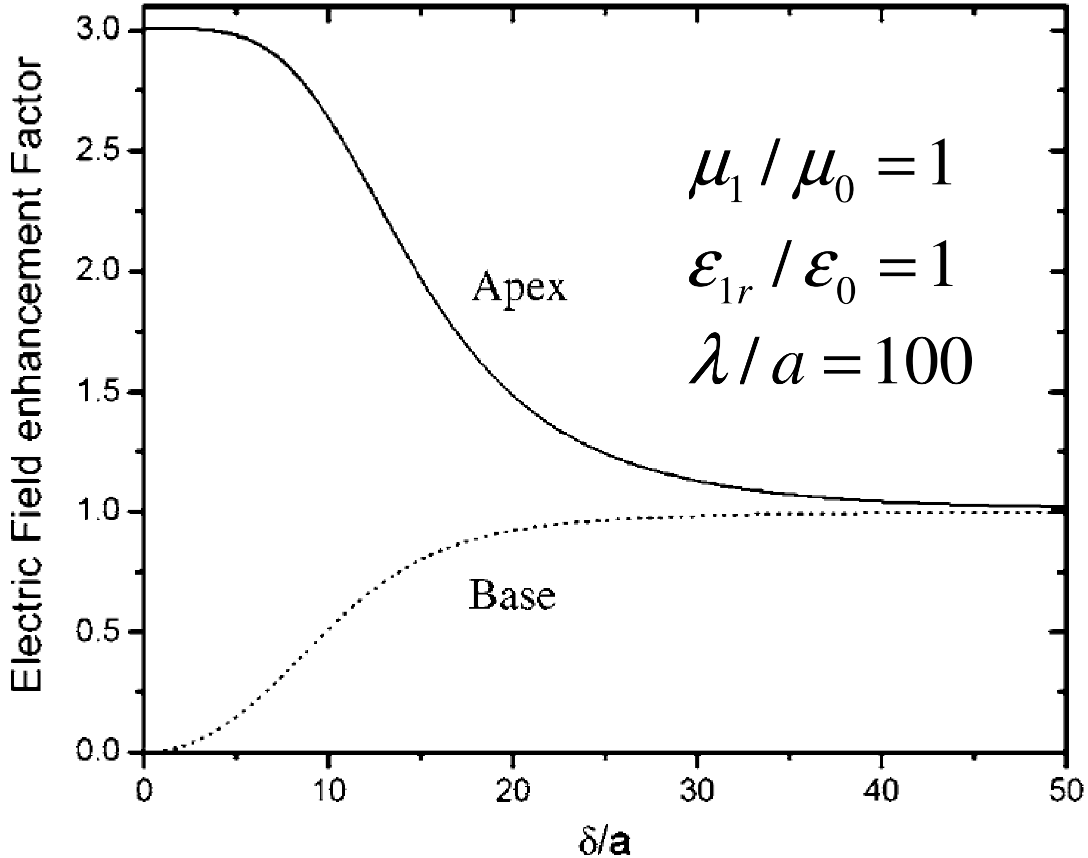
$$R = R_{max}(TM) = \left(\frac{1.33a}{\delta_s}\right) f_{protrusion}$$

RF Electric field enhancement



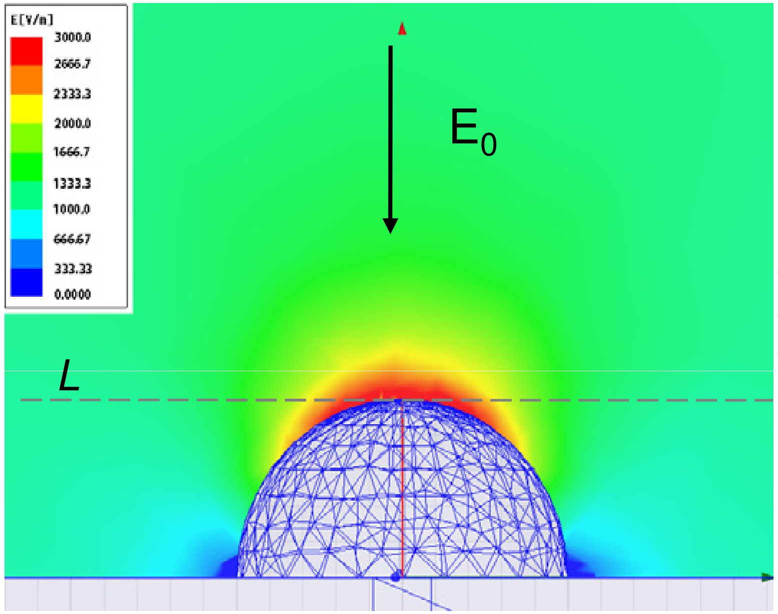
$$\beta_E = \frac{E}{E_0}$$

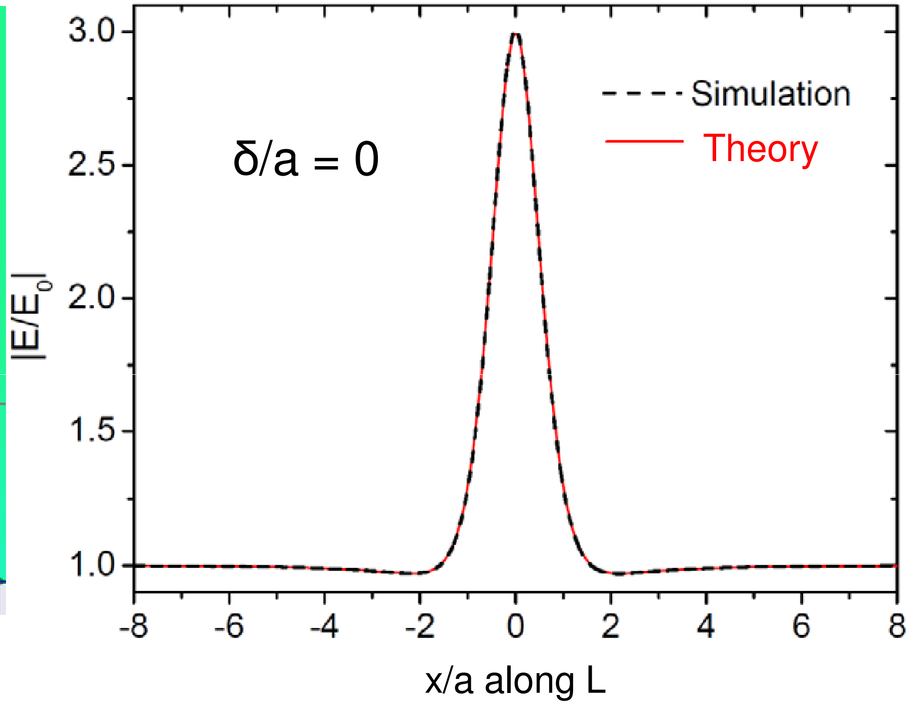
β_E = 3 at the apex
if $\delta \to 0$ [2]



[2] J. H. Jeans, *the mathematical theory of electricity and magnetism* (4th Edition, Cambridge University Press, Cambridge, 1920), p. 194.

Electric field enhancement due to hemispherical protrusion on the surface

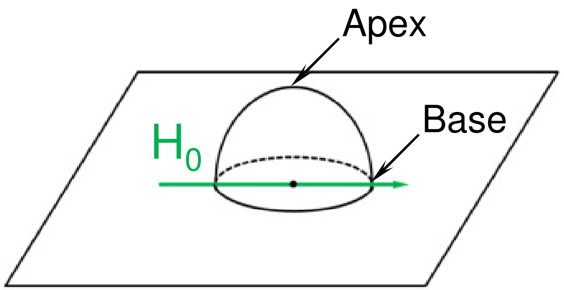




SIMULATION = MAXWELL 3D

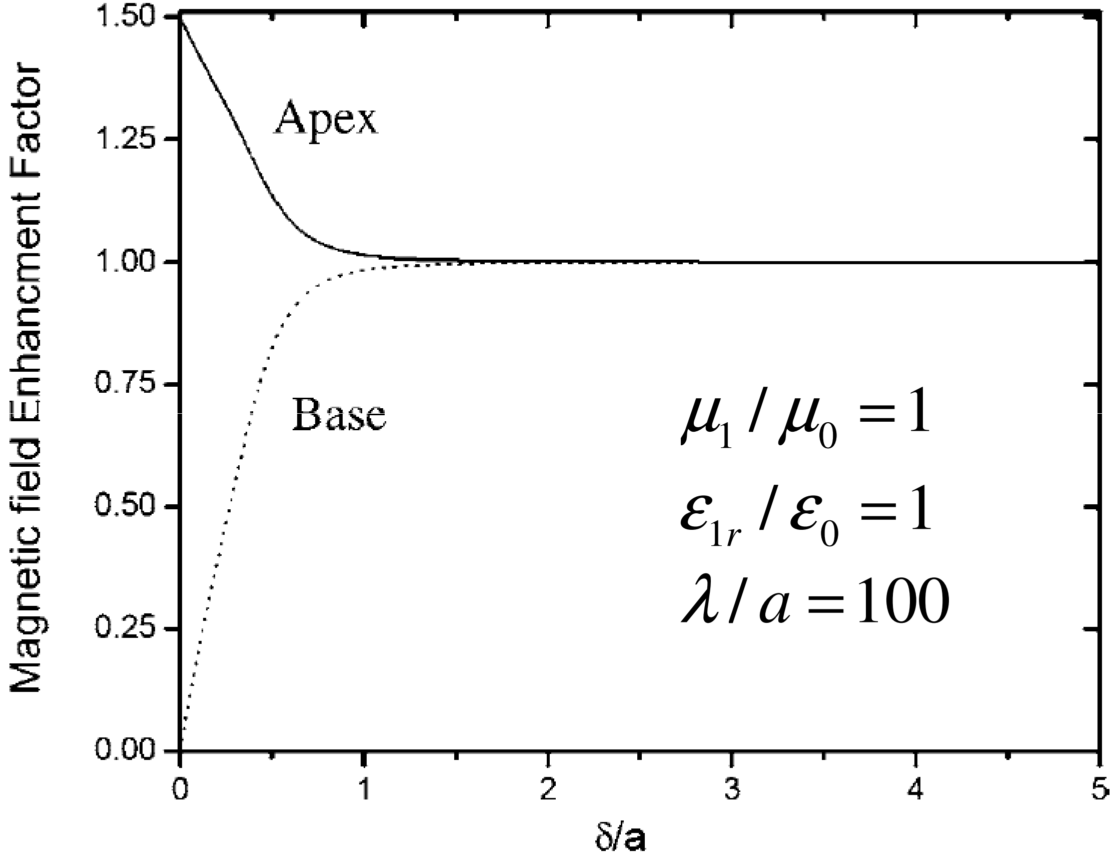
THEORY = Perturbation of Eigenfunction by Particulate

RF Magnetic field enhancement



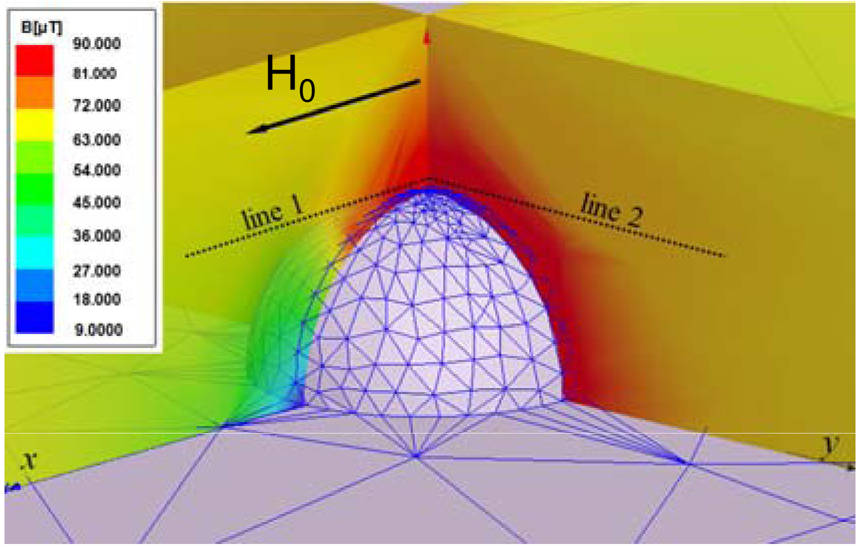
$$\beta_M = \frac{H}{H_0}$$

β_M = 1.5 at the apex
if $\delta \rightarrow 0$ [3]



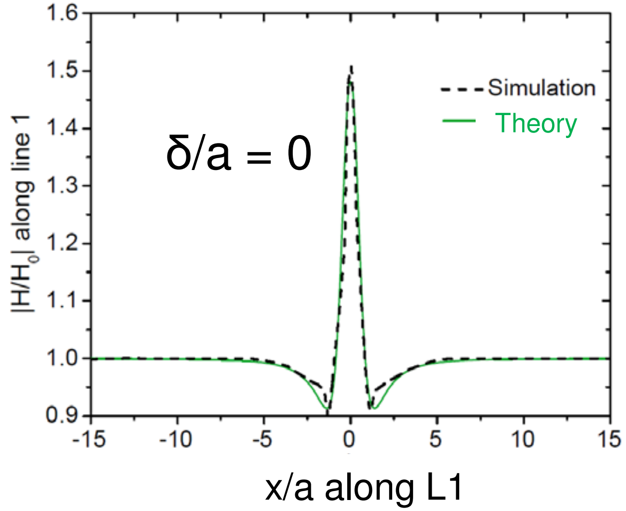
[3] A. C. Rose-Innes and E. H. Rhoderick, *Introduction to Superconductivity* (Pergamon Press, Glasgow, Scotland, 1969), p. 68.

Magnetic field enhancement due to hemispherical protrusion on the surface

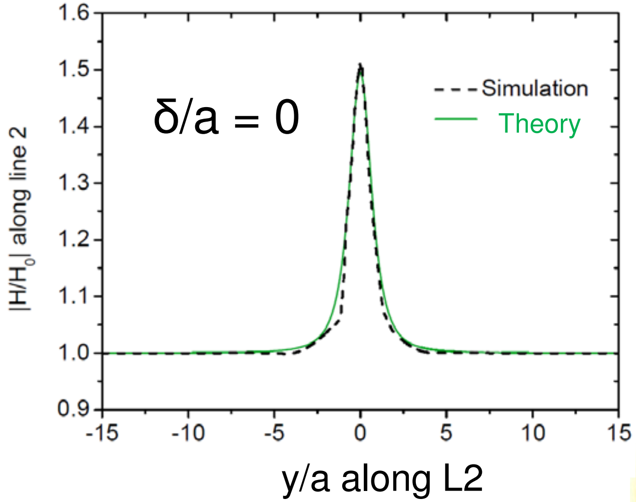


SIMULATION = MAXWELL 3D

THEORY = Perturbation of Eigenfunction by Particulate



Line 1



Line 2

Conclusion

- The RF absorption by a small hemispherical protrusion is accurately calculated for arbitrary values of (ε_r, σ, μ).
- A (non-magnetic) metallic protrusion dissipates a lot more magnetic RF energy than the electric RF energy if $\delta \ll a$.
- RF electric and magnetic field enhancements are calculated from the perturbed eigenfuctions, and confirmed by MAXWELL 3D code.
- Since the scaling laws are constructed for all ω, σ, ε, μ, the enhanced surface resistance may readily be assessed, once the distribution and composition of the surface roughness is postulated.
- Essentially calculated the scattered radiation of an arbitrary incident wave by a protrusion.

Peng Zhang, Y. Y. Lau, and R. M. Gilgenbach, "*Analysis of radio-frequency absorption and electric and magnetic enhancements due to surface roughness*", J. Appl. Phys. 105, 114908 (2009).