Evaluation of RF Power Absorption and Electric and Magnetic Field Enhancements due to Surface Roughness

Peng Zhang, Y. Y. Lau, and R. M. Gilgenbach

Department of Nuclear Engineering and Radiological Sciences
University of Michigan
Ann Arbor, MI, USA, 48109-2104

Michigan Institute for Plasma Science and Engineering
1st Annual Graduate Student Symposium
September 29, 2010

This work was supported by AFOSR, L-3 Communications, and Northrop-Grumman Corporation.
Introduction

• Surface roughness may assume many forms
  – Impurities or foreign objects
  – Manufacture tolerance, same material as the surface
  – Grain boundaries

• Surface roughness may lead to
  – Enhanced RF power loss
  – Local electric field enhancement, breakdown
  – Local magnetic field enhancement, quenching in a superconducting cavity, i.e., rapid loss of superconductivity
Outline

• Model
• RF absorption on flat metal surface
• RF absorption due to surface roughness
• Electric field enhancement due to surface roughness
• Magnetic field enhancement due to surface roughness
• Conclusion
Rough Surface

(1) Cause enhanced heating

(2) Cause local field enhancement

\[ E_{rf} \rightarrow \text{Field Emission} \rightarrow \text{Breakdown} \]

\[ H_{rf} \rightarrow \text{Quenching of Superconductor Cavity} \]
Approach

- Surface roughness is represented by a hemispherical protrusion of radius $a \ll \lambda$
  - $\varepsilon$, $\mu$, and $\sigma$ of protrusion assume arbitrary values
  - protrusion may represent foreign object or same material

- Accurately and self-consistently calculate RF electric field and RF magnetic field in presence of protrusion

- Perturbed eigenvalue gives enhanced RF loss, Perturbed eigenfunction gives RF field enhancements
Hemispherical protrusion on the surface ($a \ll \lambda_{\text{exterior}}$)

\[
(E_0, H_0) = (E_0, 0) + (0, H_0)
\]

- **E mode** $E_0$
- **M mode** $H_0$

(E mode $(E_0, 0)$)

(M mode $(0, H_0)$)
Hemispherical protrusion on the surface

The irregular geometry “hemispherical protrusion on a surface” transformed into an equivalent, but highly symmetrical problem “spherical particulate in a spherical cavity”
Hemispherical protrusion on the surface

For both $\text{TE}_{110}$ & $\text{TM}_{110}$ mode,

(A) Perturbation on Eigenvalue gives power dissipated by particulate[1]

\[
2\omega_i = \frac{P_d}{U}
\]

(B) Perturbation on Eigenfunction gives RF field enhancements

Note: (a) $(\varepsilon, \sigma, \mu)$ of protrusion may be arbitrary.
(b) Perturbed $\text{TE}_{110}$ & $\text{TM}_{110}$ mode calculated exactly, consistent with full set of Maxwell equations.

RF Power absorption due to small hemispherical protrusion on the surface

\[ \mu / \mu_0 = 1, \varepsilon_r / \varepsilon_0 = 1 \]

\[ \alpha_E = 3 \]

\[ \alpha_E \approx O(\delta) \]

\[ \alpha_E \approx O(\delta^3) \]

\[ \alpha_E \approx O(1/\delta^2) \]

\[ \alpha_H = 3 \]

\[ \alpha_H \approx O(\delta) \]

\[ \alpha_H \approx O(1/\delta^2) \]

\[ \delta = \text{skin depth} \]

\[ P_E = \alpha_E \omega \left( \frac{1}{2} \varepsilon_0 E_0^2 \right) V \]

\[ \alpha_E: \text{Electrical polarizability} \]

\[ V = \frac{2\pi}{3} a^3 = \text{Volume of protrusion} \]

\[ P_M = \alpha_H \omega \left( \frac{1}{2} \mu_0 H_0^2 \right) V \]

\[ \alpha_H: \text{Magnetic polarizability} \]
Comparison of RF Power absorption due to uncorrelated small hemispherical protrusions

I. If protrusions & flat surface of same conducting materials, $\delta = \delta_s$

$$R = \frac{\sum P_{\text{protrusion}}}{P_{\text{flat}}} = 3f_{\text{protrusion}}$$

$$f_{\text{protrusion}} = \frac{A_{\text{protrusion}}}{A_{\text{flat}}}$$

II. If protrusions are foreign objects with maximum $\alpha_E$, the maximum ohmic loss through the RF electric field is

$$R = R_{\text{max (TE)}} = \left(\frac{4a}{\delta_s}\right)f_{\text{protrusion}}$$

III. If protrusions are foreign objects with maximum $\alpha_H$, the maximum ohmic loss through the RF magnetic field is

$$R = R_{\text{max (TM)}} = \left(\frac{1.33a}{\delta_s}\right)f_{\text{protrusion}}$$
RF Electric field enhancement

\[ \beta_E = \frac{E}{E_0} \]

\[ \beta_E = 3 \text{ at the apex} \quad \text{if } \delta \to 0 \] [2]

\[ \frac{\mu_1}{\mu_0} = 1 \]

\[ \frac{\varepsilon_{1r}}{\varepsilon_0} = 1 \]

\[ \frac{\lambda}{a} = 100 \]

Electric field enhancement due to hemispherical protrusion on the surface

SIMULATION = MAXWELL 3D
THEORY = Perturbation of Eigenfunction by Particulate
RF Magnetic field enhancement

\[ \beta_M = \frac{H}{H_0} \]

\[ \beta_M = 1.5 \text{ at the apex if } \delta \rightarrow 0 \] [3]

Magnetic field enhancement due to hemispherical protrusion on the surface

\[ H_0 \]

\[ \frac{\delta}{a} = 0 \]

**Theory**

**Line 1**

**Line 2**

**SIMULATION = MAXWELL 3D**

**THEORY = Perturbation of Eigenfunction by Particulate**
Conclusion

- The RF absorption by a small hemispherical protrusion is accurately calculated for arbitrary values of $(\varepsilon_r, \sigma, \mu)$.
- A (non-magnetic) metallic protrusion dissipates a lot more magnetic RF energy than the electric RF energy if $\delta << a$.
- RF electric and magnetic field enhancements are calculated from the perturbed eigenfunctions, and confirmed by MAXWELL 3D code.
- Since the scaling laws are constructed for all $\omega, \sigma, \varepsilon, \mu$, the enhanced surface resistance may readily be assessed, once the distribution and composition of the surface roughness is postulated.
- Essentially calculated the scattered radiation of an arbitrary incident wave by a protrusion.