CONTACT RESISTANCE WITH DISSIMILAR MATERIALS: BULK CONTACTS AND THIN FILM CONTACTS

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INTRODUCTION

Our interest in contact resistance was stimulated by the recognition of its importance in our ongoing studies of the Z-pinch, high power microwave generation, triple point junctions, field emitters, and heating phenomenology. In learning the subject, we were always referred to the classical reference of Holm [1].

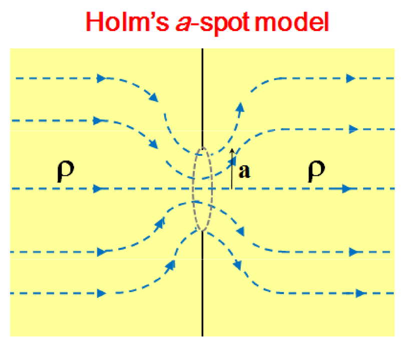
Holm's *a*-spot theory gives the electrical contact resistance of a circular constriction between two contacting surfaces. Implicit in the theory of Holm are several assumptions: (A) the *a*-spot has a zero thickness, i.e., zero axial length in the direction of current flow, (B) the current channel is made of the same material, e.g., the effects of contaminants have been ignored, and (C) the contact members are bulk conductors, whose dimensions transverse to the current flow are infinite.

Here, we present a vast generalization of the conventional theory of bulk contacts [2] and thin film contacts [3], by relaxing assumptions (A), (B) and (C) mentioned above.

h = 0

Radius a

Holm's *a*-spot model



$$R=\frac{\rho}{4a}+\frac{\rho}{4a}$$

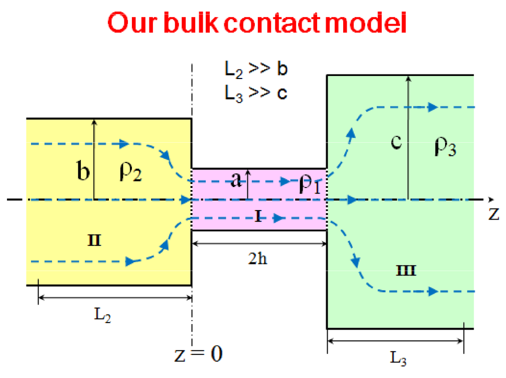
h > 0,

Radius a≠ b ≠ c,

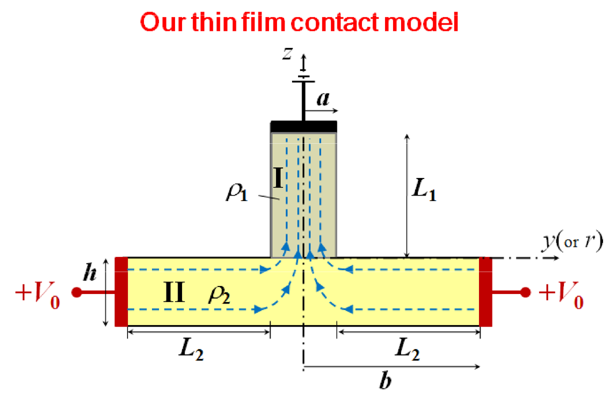
Resistivity $\rho_1 \neq \rho_2 \neq \rho_3$

Cylindrical or Cartesian

Our bulk contact model



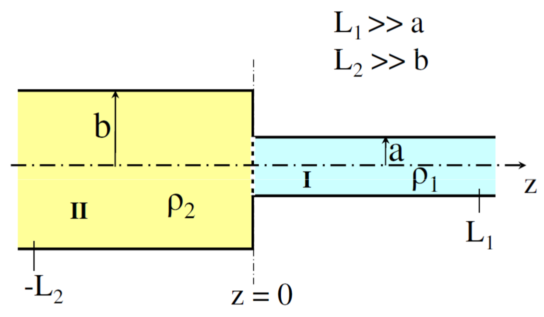
Our thin film contact model



BULK CONTACTS

Interface Resistance with Dissimilar Materials

A. Cartesian semi-infinite channel



Cartesian or Cylindrical

Laplace's equation

$$\Phi_+(y,z)=\sum_{n=0}^{\infty}A_n\cos\left(\frac{n\pi y}{a}\right)e^{-\frac{n\pi z}{a}}-E_{+\infty}z,\qquad z>0,y\in(0,a),$$

$$\Phi_-(y,z)=\sum_{n=1}^{\infty}B_n\cos\left(\frac{n\pi y}{b}\right)e^{+\frac{n\pi z}{b}}-E_{-\infty}z,\qquad z<0,y\in(0,b),$$

Boundary conditions

$$\Phi_+=\Phi_-,\quad z=0,\ y\in(0,a);\qquad \frac{1}{\rho_1}\frac{\partial\Phi_+}{\partial z}=\frac{1}{\rho_2}\frac{\partial\Phi_-}{\partial z},\quad z=0,\ y\in(0,a);\qquad \frac{\partial\Phi_-}{\partial z}=0,\quad z=0,\ y\in(a,b)$$

⇒ Potential profile & interface resistance for arbitrary *a*, *b*, ρ_1, ρ_2

Exact Solution

$$R=\underbrace{\frac{\rho_2L_2}{2b\times W}}_{\text{Bulk}}+\underbrace{\frac{\rho_2}{4\pi W}\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_1L_1}{2a\times W}}_{\text{Bulk}}$$

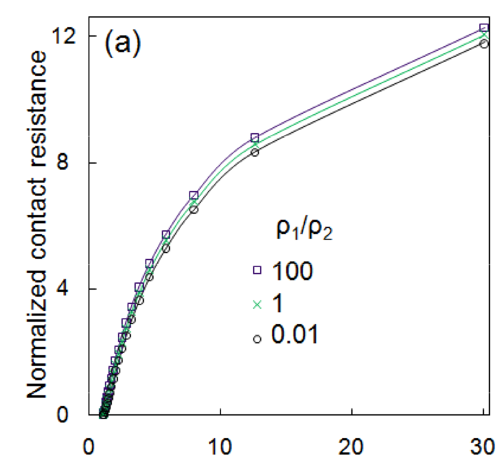
Scaling laws

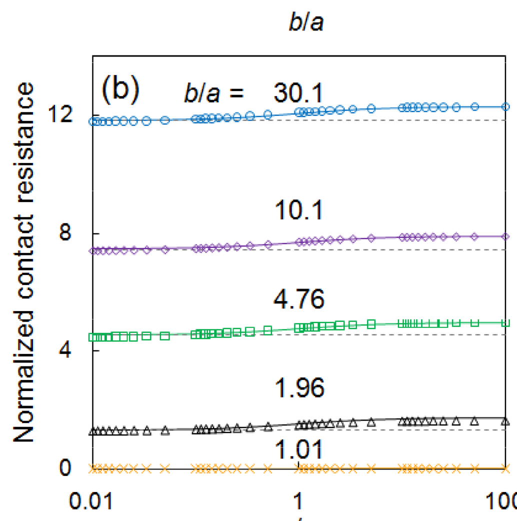
$$\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)\cong\bar{R}_{c0}\left(\frac{b}{a}\right)\Bigg|_{LTZ}+0.2274\times\left(\frac{2\rho_1}{\rho_1+\rho_2}\right)\times g\left(\frac{b}{a}\right)$$

$$\bar{R}_{c0}(b/a)\Big|_{LTZ}=4\ln(2b/\pi a)+4\ln(\pi/2)\times f(b/a),$$

$$f(b/a)=0-0.03250(a/b)+1.06568(a/b)^2-0.24829(a/b)^3+0.21511(a/b)^4,$$

$$g(b/a)=1-1.2281(a/b)^2+0.1223(a/b)^4-0.2711(a/b)^6+0.3769(a/b)^8,$$





Symbols: Exact theory
Solid lines: Scaling laws

B. Cylindrical semi-infinite channel

Laplace's equation

$$\Phi_+(r,z)=A_0+\sum_{n=1}^{\infty}A_nJ_0(\alpha_nr)e^{-\alpha_nz}-E_{+\infty}z,\qquad z>0,r\in(0,a);$$

$$\Phi_-(r,z)=\sum_{n=1}^{\infty}B_nJ_0(\beta_nr)e^{+\beta_nz}-E_{-\infty}z,\qquad z<0,r\in(0,b).\qquad J_1(\alpha_na)=J_1(\beta_nb)=0$$

Boundary conditions

$$\Phi_+=\Phi_-,\quad z=0,\ r\in(0,a);\qquad \frac{1}{\rho_1}\frac{\partial\Phi_+}{\partial z}=\frac{1}{\rho_2}\frac{\partial\Phi_-}{\partial z},\quad z=0,\ r\in(0,a);\qquad \frac{\partial\Phi_-}{\partial z}=0,\quad z=0,\ r\in(a,b)$$

⇒ Potential profile & interface resistance for arbitrary *a*, *b*, ρ_1, ρ_2

Exact Solution

$$R=\underbrace{\frac{\rho_2L_2}{\pi b^2}}_{\text{Bulk}}+\underbrace{\frac{\rho_2}{4a}\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_1L_1}{\pi a^2}}_{\text{Bulk}}$$

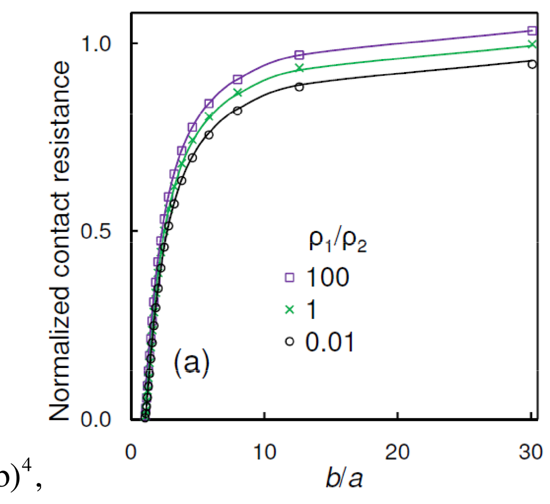
Scaling laws

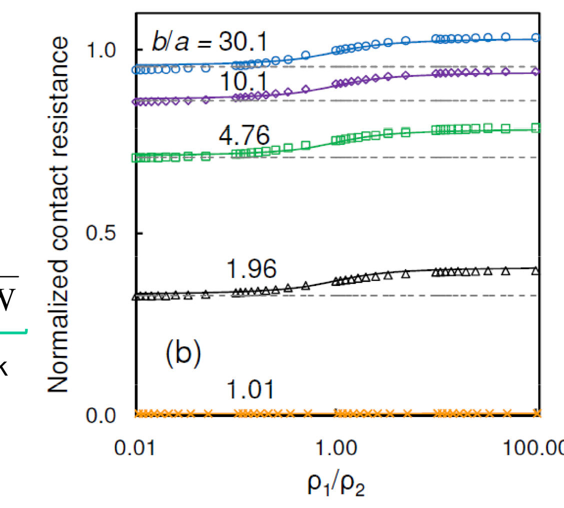
$$\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)\cong\bar{R}_{c0}\left(\frac{b}{a}\right)\Bigg|_{Timsit}+\frac{\Delta}{2}\times\left(\frac{2\rho_1}{\rho_1+\rho_2}\right)\times g\left(\frac{b}{a}\right)$$

$$\bar{R}_{c0}(b/a)\Big|_{Timsit}=1-1.41581(a/b)+0.06322(a/b)^2+0.15261(a/b)^3+0.19998(a/b)^4,$$

$$g(b/a)=1-0.3243(a/b)^2-0.6124(a/b)^4-1.3594(a/b)^6+1.2961(a/b)^8,$$

$$\Delta=32/3\pi^2-1=0.08076$$





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Total Resistance of Composite Channel

$$R_{Cartesian}=\underbrace{\frac{\rho_2L_2}{2b\times W}}_{\text{Bulk}}+\underbrace{\frac{\rho_2}{4\pi W}\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_1\times 2h}{2a\times W}}_{\text{Bulk}}+\underbrace{\frac{\rho_3}{4\pi W}\bar{R}_c\left(\frac{c}{a},\frac{\rho_1}{\rho_3}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_3L_3}{2c\times W}}_{\text{Bulk}}$$

$$R_{cylindrical}=\underbrace{\frac{\rho_2L_2}{\pi b^2}}_{\text{Bulk}}+\underbrace{\frac{\rho_2}{4a}\bar{R}_c\left(\frac{b}{a},\frac{\rho_1}{\rho_2}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_1\times 2h}{\pi a^2}}_{\text{Bulk}}+\underbrace{\frac{\rho_3}{4a}\bar{R}_c\left(\frac{c}{a},\frac{\rho_1}{\rho_3}\right)}_{\text{Interface}}+\underbrace{\frac{\rho_3L_3}{\pi c^2}}_{\text{Bulk}}$$

THIN FILM CONTACTS

A. Cartesian thin film contact

Exact Solution

$$R=\underbrace{\frac{\rho_2L_2}{2h\times W}}_{R_{II}}+\underbrace{\frac{\rho_2}{4\pi W}\bar{R}_c\left(\frac{a}{b},\frac{a}{h},\frac{\rho_1}{\rho_2}\right)}_{\text{Contact resistance } R_c}+\underbrace{\frac{\rho_1L_1}{2a\times W}}_{R_I}$$

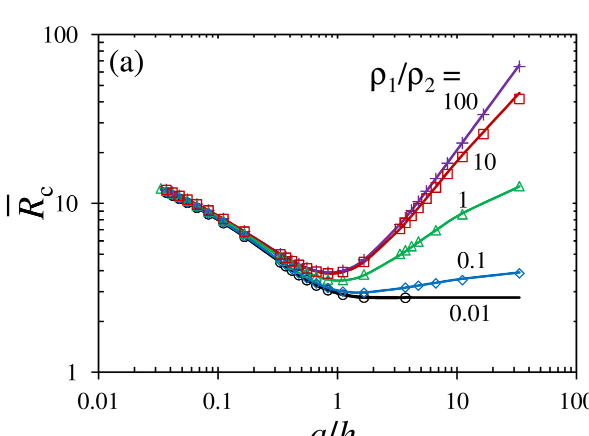
Scaling laws

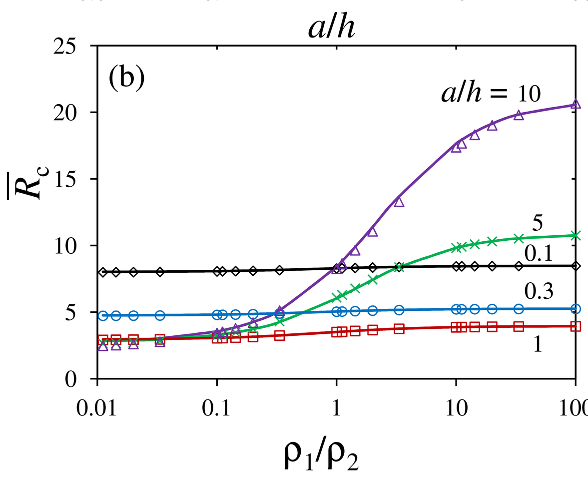
$$\bar{R}_c(a/h,\rho_1/\rho_2)\cong\bar{R}_{c0}(a/h)+\frac{\Delta(a/h)}{2}\times\frac{2\rho_1}{\rho_1+\beta(a/h)\rho_2},$$

$$\bar{R}_{c0}(a/h)=\bar{R}_c(a/h)\Big|_{\rho_1/\rho_2\to0}=2\pi a/h-4\ln\left[\sinh(\pi a/2h)\right],$$

$$\Delta(a/h)=\begin{cases}0.5346(a/h)^2+0.0127(a/h)+0.4548, & 0.03\le a/h\le1;\\ 0.0147x^6-0.0355x^5+0.1479x^4+0.4193x^3\\ +1.1163x^2+0.9970x+1, & x=\ln(a/h),1<a/h\le30,\end{cases}$$

$$\beta(a/h)=-0.0003(a/h)^2+0.1649(a/h)+0.6727,\quad 0.03\le a/h\le30.$$





Symbols: Exact theory
Solid lines: Scaling laws

B. Cylindrical thin film contact

Exact Solution

$$R=\underbrace{\frac{\rho_2}{2\pi h}\ln\left(\frac{b}{a}\right)}_{R_{II}}+\underbrace{\frac{\rho_2}{4a}\bar{R}_c\left(\frac{a}{b},\frac{a}{h},\frac{\rho_1}{\rho_2}\right)}_{\text{Contact resistance } R_c}+\underbrace{\frac{\rho_1L_1}{\pi a^2}}_{R_I}$$

Scaling laws

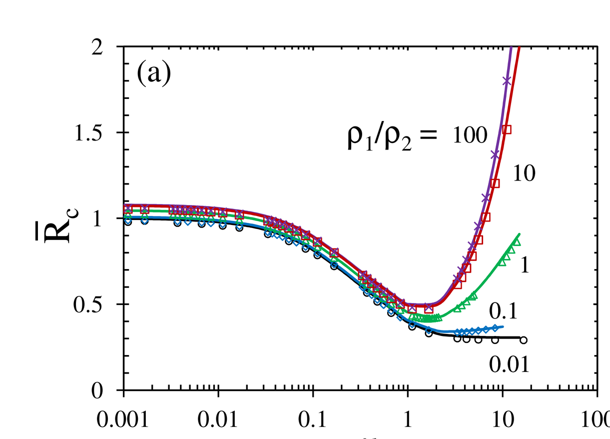
$$\bar{R}_c(a/h,\rho_1/\rho_2)\cong\bar{R}_{c0}(a/h)+\frac{\Delta(a/h)}{2}\times\frac{2\rho_1}{\rho_1+\beta(a/h)\rho_2},$$

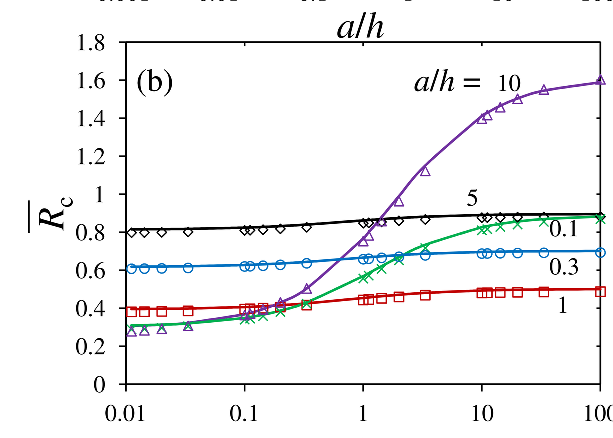
$$\bar{R}_{c0}(a/h)=\bar{R}_c(a/h)\Big|_{\rho_1/\rho_2\to0}$$

$$=\begin{cases}1-2.2968(a/h)+4.9412(a/h)^2-6.1773(a/h)^3\\ +3.811(a/h)^4-0.8836(a/h)^5, & 0.001\le a/h\le1;\\ 0.295+0.037(h/a)+0.0595(h/a)^2, & 1<a/h<10,\end{cases}$$

$$\Delta(a/h)=\begin{cases}0.0184(a/h)^2+0.0073(a/h)+0.0808,\\ \qquad 0.001\le a/h\le1;\\ 0.0409x^4-0.1015x^3+0.265x^2-0.0405x\\ +0.1065, \quad x=\ln(a/h),1<a/h<10,\end{cases}$$

$$\beta(a/h)=0.0016(a/h)^2+0.0949(a/h)+0.6983,\quad 0.001\le a/h<10.$$





Symbols: Exact theory
Solid lines: Scaling laws

CONCLUSIONS

- Simple, accurate analytical scaling laws of contact resistance with dissimilar materials were constructed for both bulk and thin film contacts. They were validated against known limiting cases, experiments, and numerical simulations
- Interface resistance of bulk contacts depends mainly on the electrical resistivity of the main channel (ρ_2, ρ_3); it is insensitive to the resistivity of the contact region (ρ_1)
- For fixed ρ_1/ρ_2, thin film contact resistance primarily depends on a/h, as long as either L_2 >> a or L_2 >> h
- The minimum thin film contact resistance occurs at *a/h* ~ 1, regardless of ρ_1 and ρ_2

[1] Holm, *Electric Contacts: Theory and Application*, Springer-Verlag, NY (1967); A. M. Rosenfeld and R. S. Timsit, *Quart Appl. Math.*, vol. 39, p. 405, 1981.
[2] Y. Y. Lau and W. Tang, *J. Appl. Phys.* **105**, 124902 (2009); M. R. Gomez et al., *Appl. Phys. Lett.* **95**, 072103 (2009); P. Zhang et al., *J. Appl. Phys.* **108**, 044914 (2010).
[3] P. Zhang et al., *Appl. Phys. Lett.* **97**, 204103 (2010); *J. Appl. Phys.* **109**, 124910 (2011); Proc. of the 57th IEEE Holm Conf. on Electrical Contacts (2011).

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