Effects of Random Circuit Fabrication Errors on Small Signal Gain in a Traveling Wave Tube

I. M. Rittersdorf

University of Michigan, Ann Arbor, MI 48109, USA

T. M. Antonsen, Jr.

University of Maryland, College Park, MD 20742, USA

D. Chernin

Science Applications International Corporation, McLean, VA 22102, USA

Y. Y. Lau

University of Michigan, Ann Arbor, MI 48109, USA

This work was supported by AFOSR, ONR, L-3 Communications Electron Devices and Northrop Grumman Corporation.

Motivations

– Random circuit errors affect TWT performance, manufacturing yield, and cost. This problem is increasingly serious at millimeter and sub-millimeter wavelengths.

– Seek to derive scaling laws for the ensemble-averaged gain and phase for TWT with random axial variations in circuit parameters.

– To extend existing work into regimes with non-synchronous beam velocities and to include the effects of the Pierce "space-charge" term.

Continuum Model of TWT

- Governing third-order differential equation in $x = \omega z/v_b$ according to Pierce's 3-wave theory

$$\frac{d^3 f(x)}{dx^3} + jC(b - jd)\frac{d^2 f(x)}{dx^2} + 4QC^3 \frac{df(x)}{dx} + jC\left(4QC^3(b - jd) + C^2\right) f(x) = 0$$

where

$f(x) = e^{jx} E_{rf}(x)$ is Pierce's 3-wave solution;

b is the mismatch between beam and circuit phase velocities;

C is Pierce's gain parameter, assumed to be a constant;

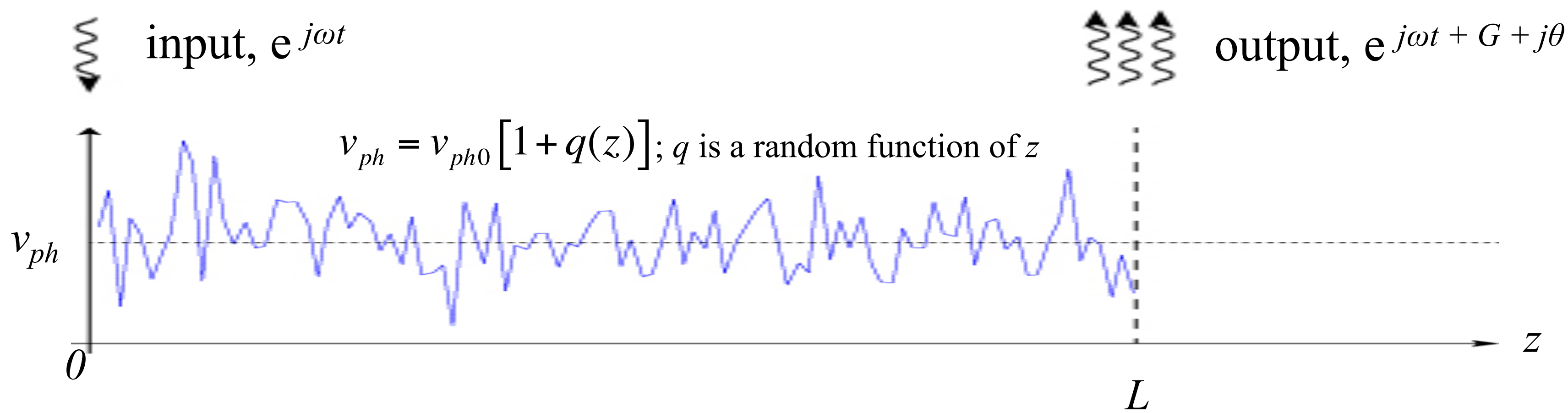
d is the cold tube circuit loss rate, assumed zero here;

QC is Pierce's "space charge" term.

Concentrate mainly on random variations in $b(x)$

Random Circuit Fabrication Errors

- Assume circuit phase velocity, v_p, with a random error represented by $q(x)$.



Three Wave

$$\left(\frac{\partial}{\partial z} + \frac{i\omega}{v_0}\right)^2 s_1 = \varepsilon_1$$

[Beam's response to EM wave] (2 beam modes)

$$\left(\frac{\partial}{\partial z} + \frac{i\omega}{v_{ph}}\right)\varepsilon_1 = C^3 s_1$$

[Excitation of EM wave by AC current of beam] (one forward circuit mode)

Modifications in Output Gain and Phase

$$\left| \frac{f''(x) + 4QC^3 f(x)}{f_0''(x) + 4QC^3 f_0(x)} \right| = e^{G_1 + j\theta_1}$$

$f_0(x)$ = error-free, 3-wave solution

G_1 and θ_1 depend on:

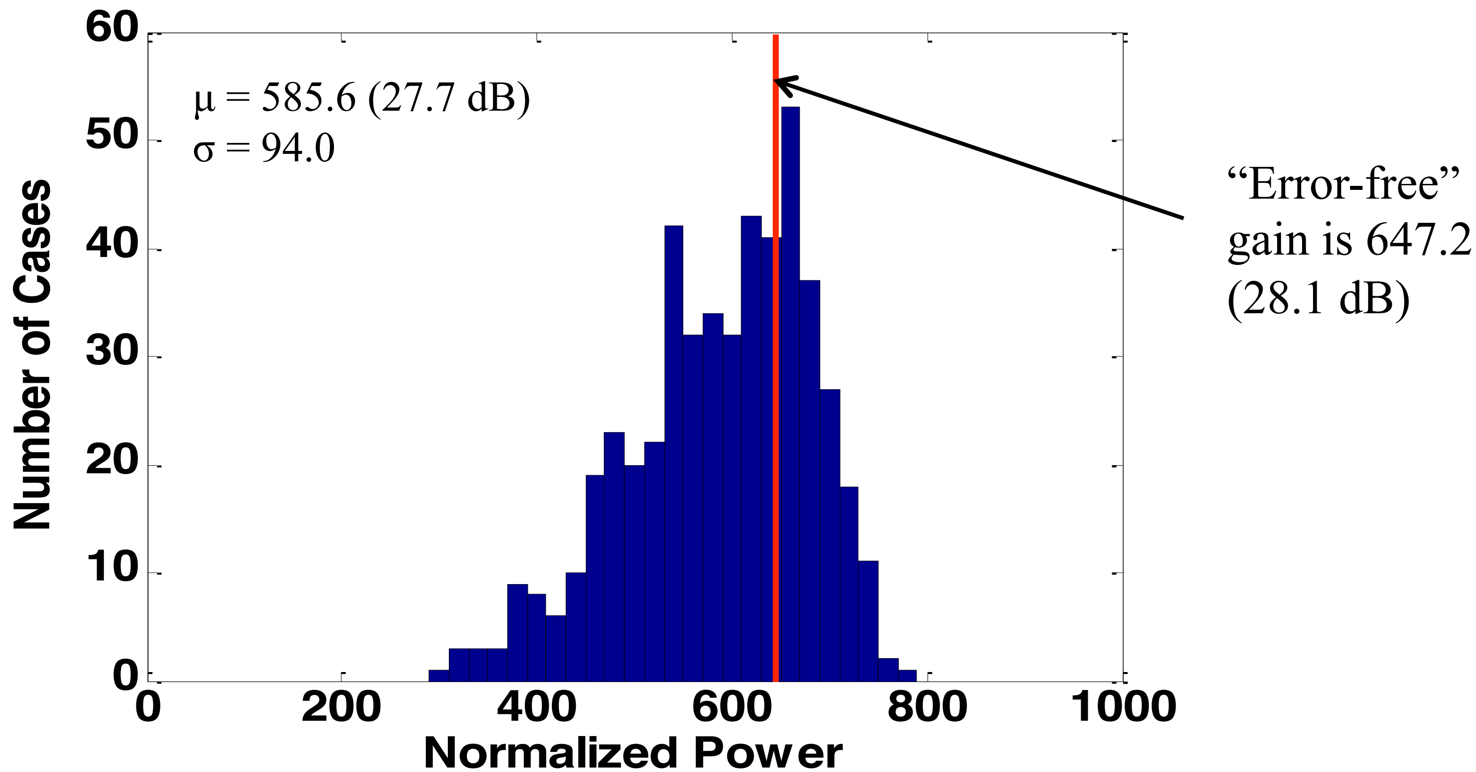
tube length (x)

correlation length ($\Delta = x/N$)

standard deviation in b (σ_b)

Statistics of 500 Runs

Power distribution at x = 100 for $b_0 = d = 0$, $C = 0.05$
$\sigma_b = 1.7$ (corresponds to $\sigma_q = 0.085$)

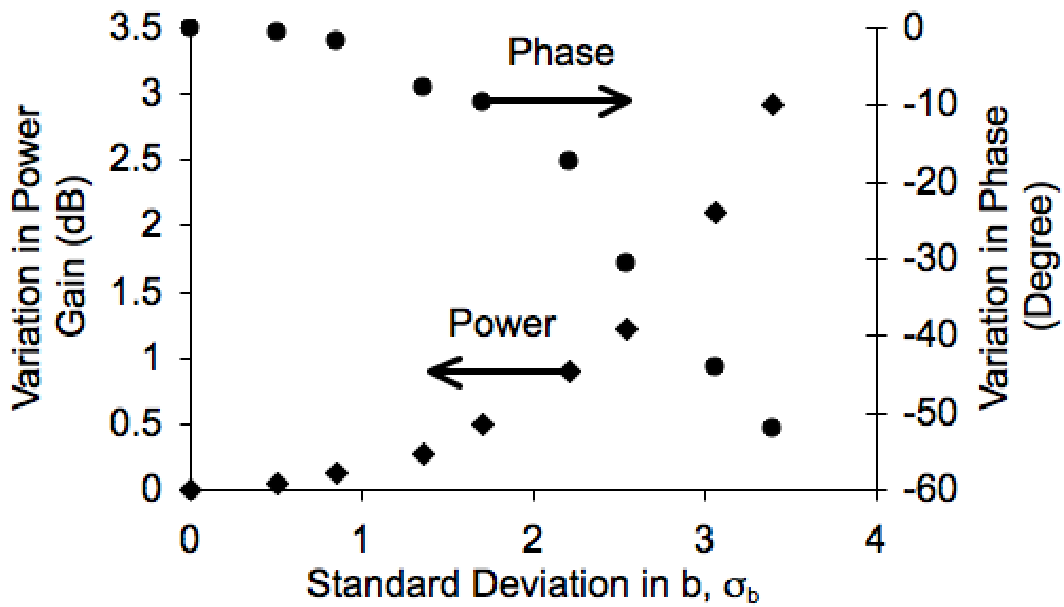


Note: In significant number of cases the gain is actually higher (!) than the "error-free" case.

[1] Pengvenich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

Statistics of 500 runs (cont'd)

Variations at x = 100 for $b_0 = d = 0$, $C = 0.05$



The variations in gain and phase appear quadratic in σ_b. Scaling law is to be derived (in this paper).

Pengvenich *et. al., IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

Two Analytic Approaches

- Perturbative analysis. Linear theory carried to second order in $q(x)$

$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{1}{2}\sigma_b^2 \Delta \int_0^x P(x,s)\,ds$$

$P(x,s)$ depends only on error-free, 3-wave solution.

- Riccati analysis. Nonlinear formulation of wavenumber, for a *single wave*

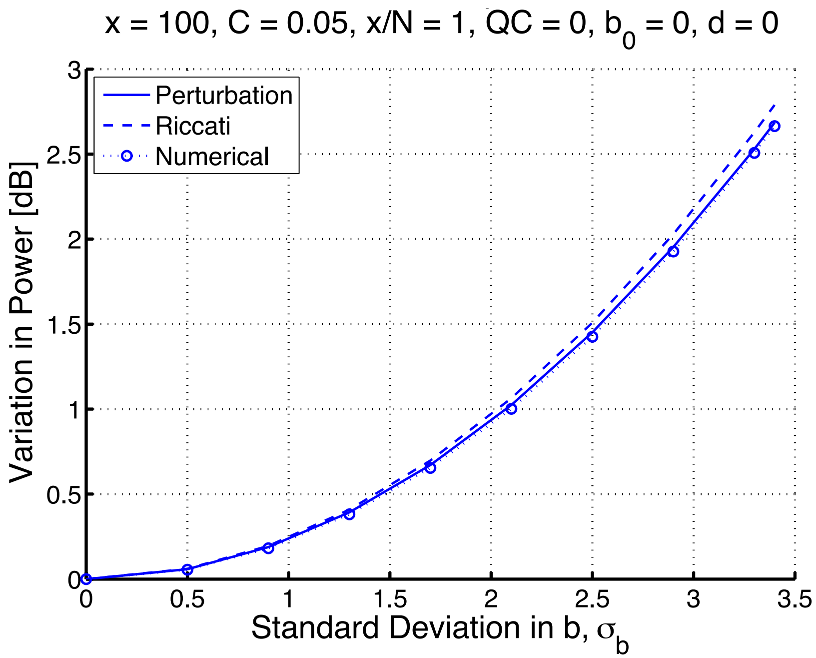
$$\left\langle G_1(x) + j\theta_1(x) \right\rangle = -\frac{\lambda}{2}\left(\frac{C}{1+Cb_0}\right)^2 x\sigma_b^2 \Delta$$

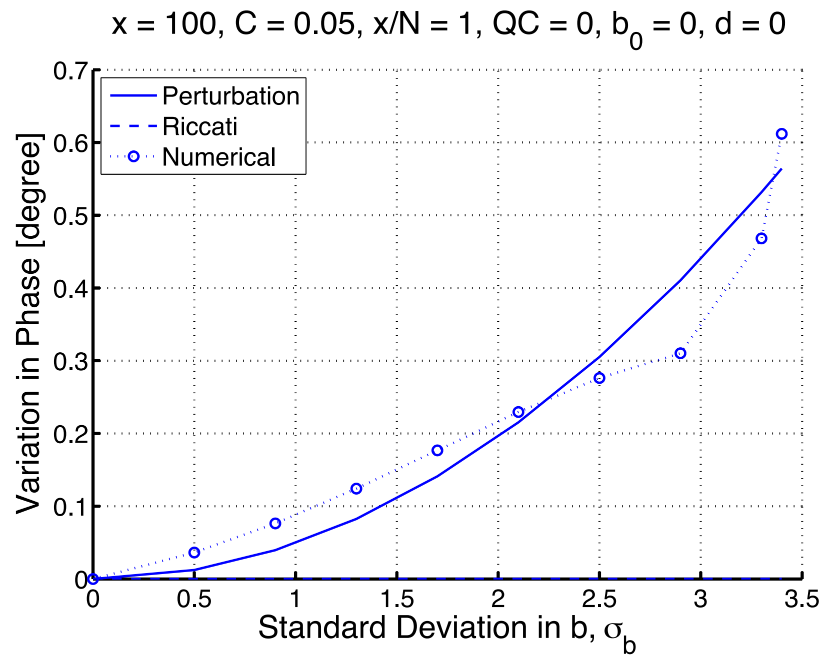
where λ is a complex constant that depends on the velocity mismatch parameter, b_0.

Note: $\left\langle \theta_1(x) \right\rangle = 0 \;\; (!), \;\; (b_0 = 0)$

*Previous work focused only on standard deviations

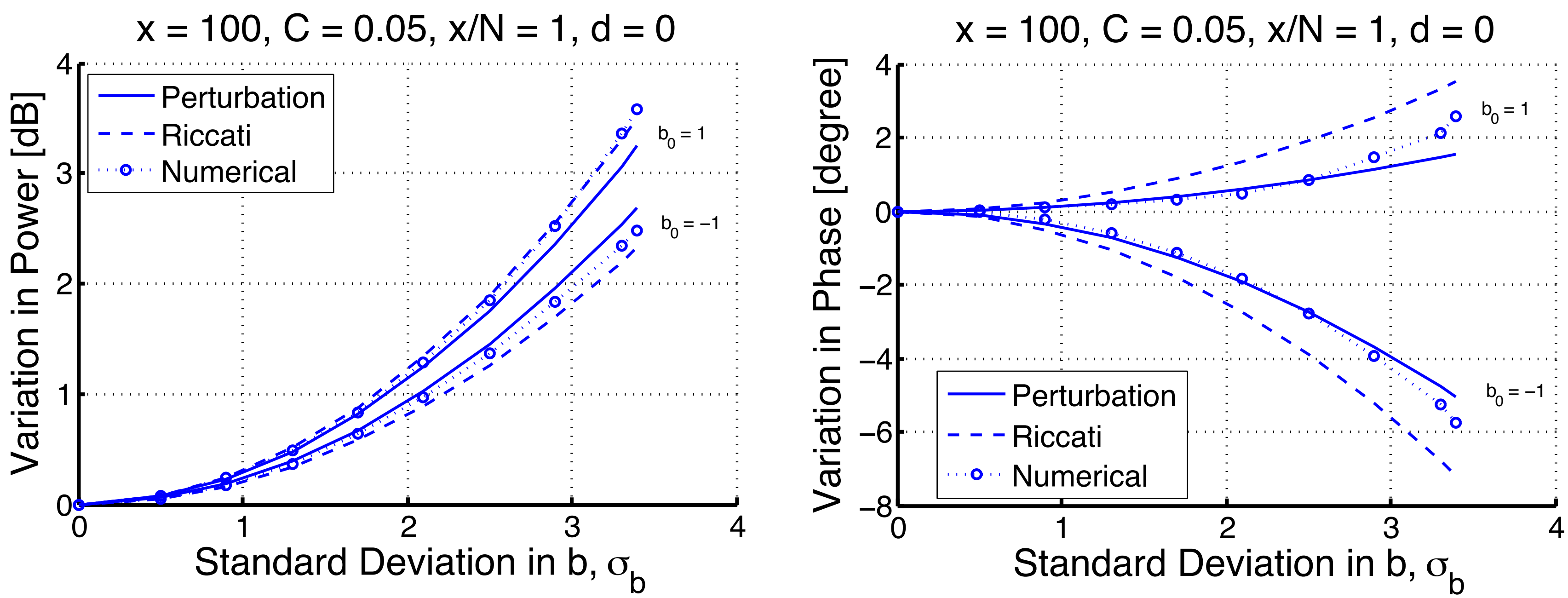
Perturbation, Riccati Analysis





- Perturbation analysis shows good agreement with numerical solution for gain over a wide range of parameters.
- Perturbation analysis yields phase variation close to zero, similar to the Riccati analysis. Numerical solutions also show small variations in phase.

Perturbation, Riccati Analysis

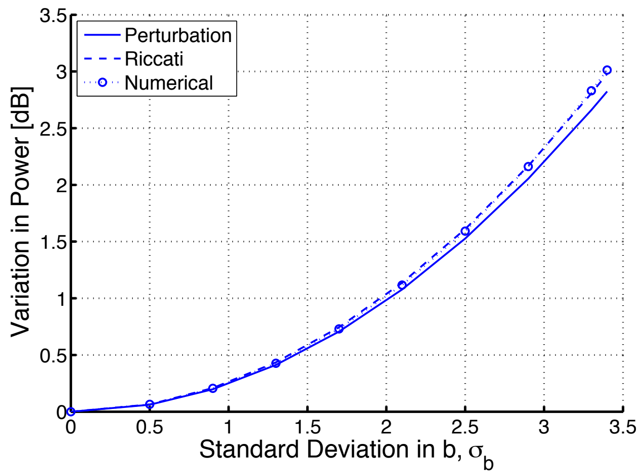


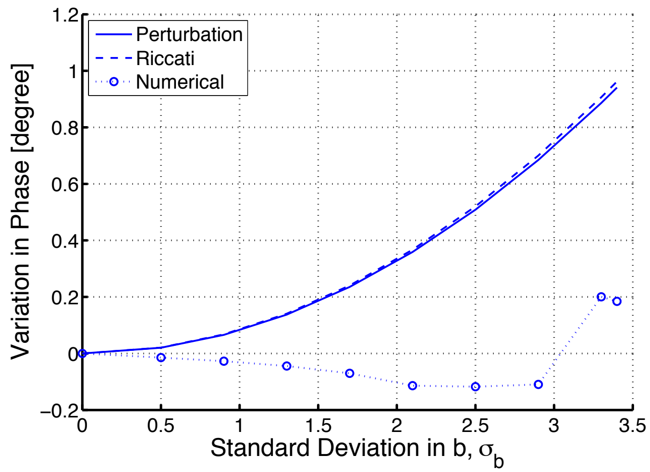
- All three methods are in agreement for non-synchronous beam velocities
- Perturbation analysis more accurate than Riccati analysis as expected due to Riccati analysis only considering a single wave

Perturbation, Riccati Analysis

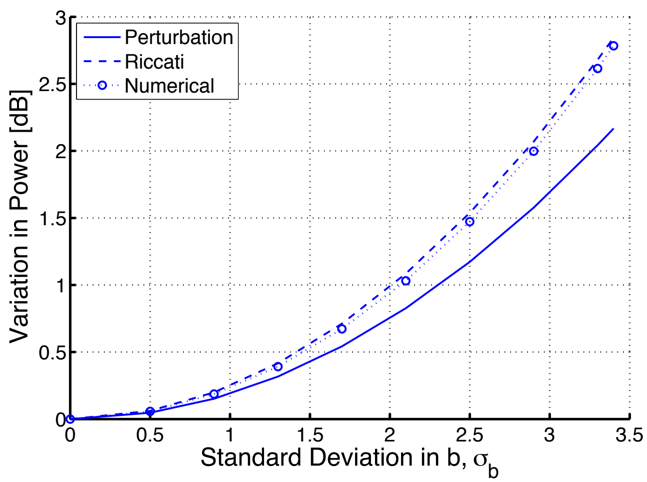
Nonzero QC

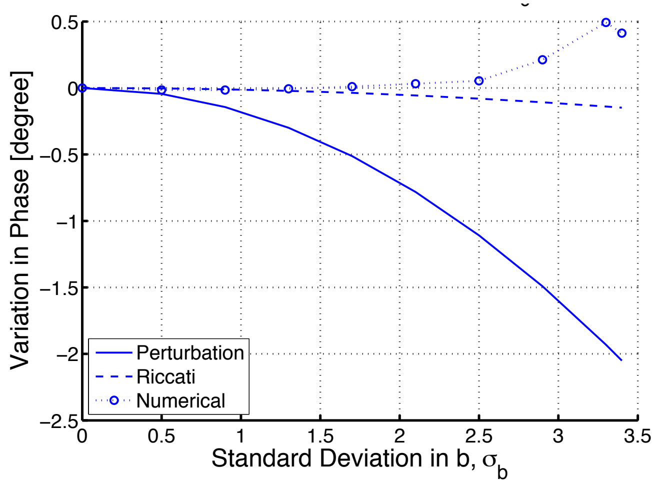
QC = 0.15





QC = 0.35





Standard Deviation Analysis

$$\sigma_{Gb} = S_{Gb}\sigma_b \qquad S_{Gb} = \sqrt{\frac{x}{N}}\sqrt{\int_0^x ds\left|g_{br}(x,s)\right|^2}$$

$$\sigma_{\theta b} = S_{\theta b}\sigma_b \qquad S_{\theta b} = \sqrt{\frac{x}{N}}\sqrt{\int_0^x ds\left|g_{bi}(x,s)\right|^2}$$

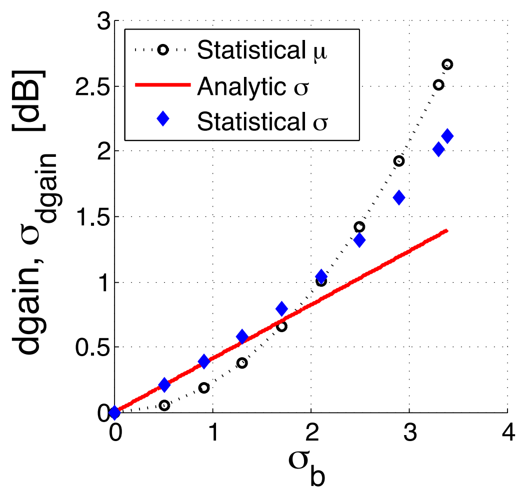
$$g_b = -jC\left(4QC^3 f_0(s) + a_0(s)\right)a_0(x-s)/a_0(x)$$

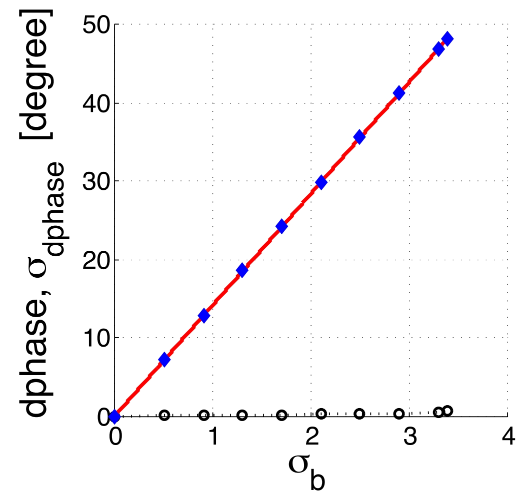
Extension of [1] to include "space charge" term

[1] Pengvenich, Chernin, Lau, Luginsland, Gilgenbach, *IEEE Trans. Electron Devices*, vol. 55, no. 3, pp. 916-924 (2008)

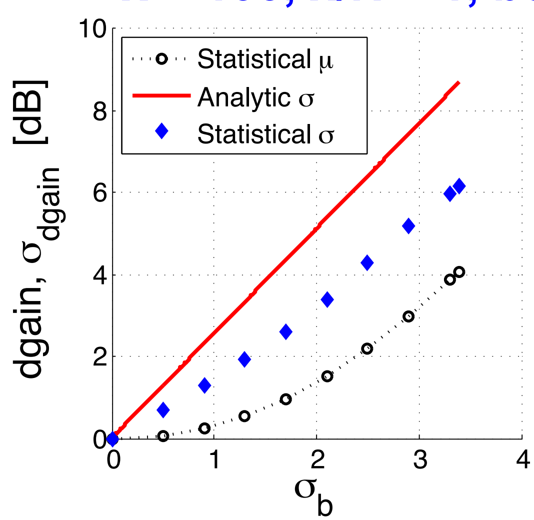
Standard Deviation Analysis

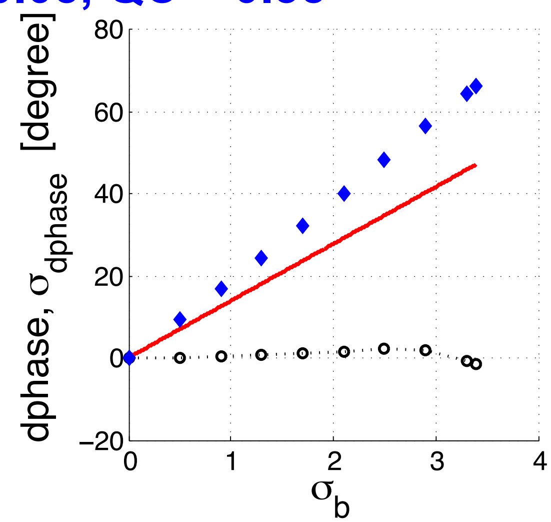
x = 100, x/N = 1, b0 = 0, C = 0.05, QC = 0





x = 100, x/N = 1, b0 = 0, C = 0.05, QC = 0.35





Four-Wave Analysis[2]

$$\left(\frac{\partial}{\partial z} + j\beta_e\right)^2 s = \beta_e^2 E$$

$$\left(\frac{\partial^2}{\partial z^2} + \beta_p^2\right) E = -2\beta_p \beta_e C^3 s$$

Second equation includes reverse propagating waves.

Focus on random variations in b (i.e., in β_p).

[2] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Four-Wave Results

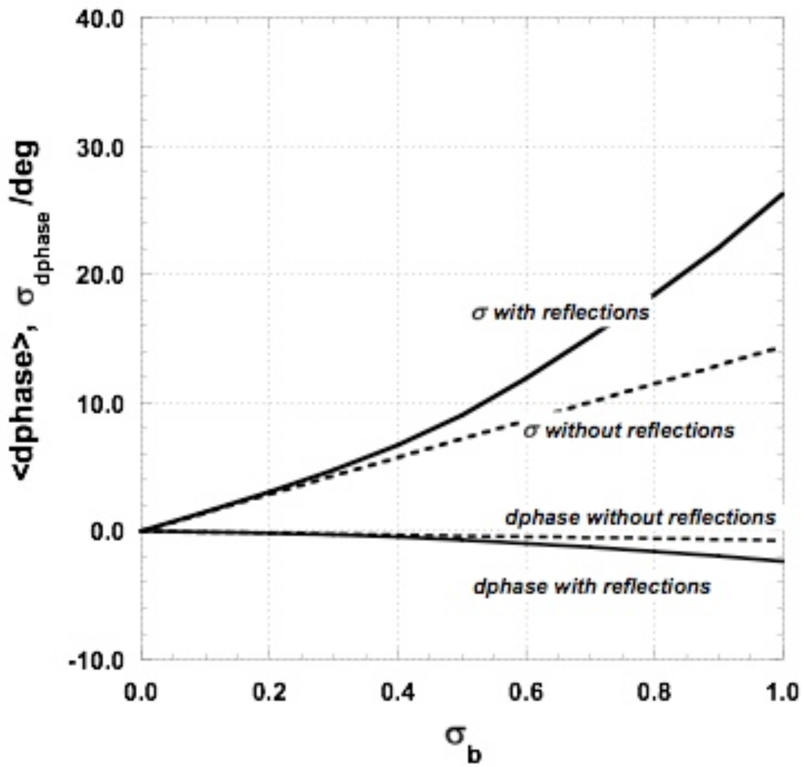


Figure 9: Departures from the error free values of small signal phase and the standard deviations of those departures vs. σ_b. Solid lines indicate results including reflections (4th order model); dashed lines indicate results omitting reflections (3rd order model). $C = 0.05$, $\overline{b} = 0$, $x_N = 100$, $N = 100$. The error free values of phase are -113.6° and -112.1° with and without reflections, respectively.

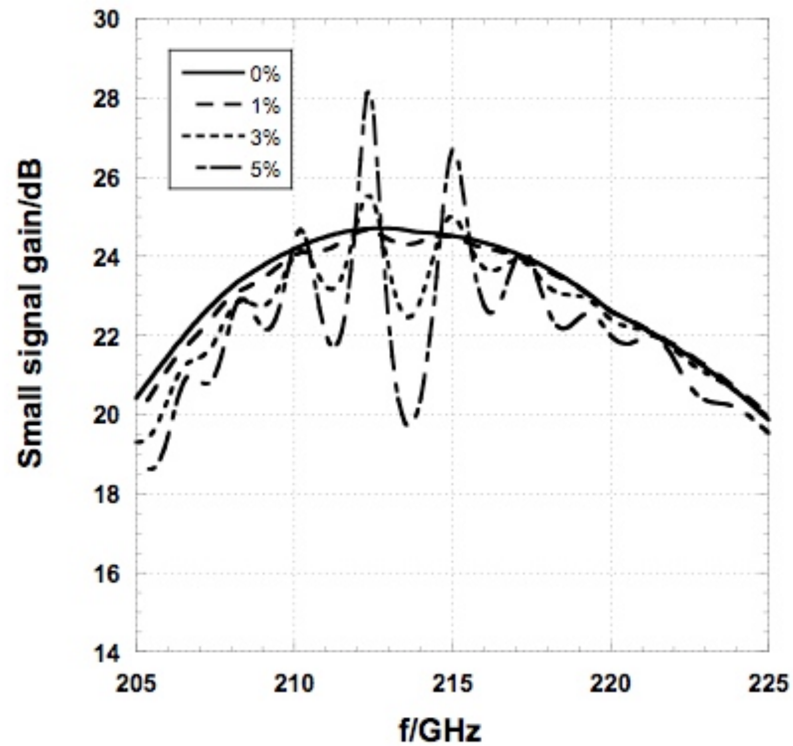


Figure 13: Small signal gain vs. frequency for different values of standard deviation of the circuit pitch distribution.

[2] Chernin, Rittersdorf, Lau, Antonsen, and Levush, IEEE Trans. Electron Devices, vol. 59, 1542 (2012).

Conclusion

- Deviation in small signal gain is found to be quadratic in σ_b, the standard deviation in Pierce's velocity mismatch parameter b.
- Good agreement is found between perturbative analytic theory of three waves and numerical computation in the absence of space charge effects.
- Effects of reverse propagating wave (four-wave theory) developed. Effects on gain and phase are significant.
- Remaining problems:

  Higher gain than error-free tubes occurs in a significant fraction of runs.

  TWT oscillations caused by reflected waves from random errors?