Boundary Conditions in a Two-dimensional, Axisymmetric, Direct Kinetic Hall Thruster Simulation

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Introduction

- A two-dimensional, axisymmetric, hybrid-direct kinetic (DK) simulation is under development for use in Hall thruster applications.
- The simulation employs a deterministic, two-dimensional Vlasov solver coupled with a quasi one-dimensional electron fluid solver to model the plasma discharge in a Hall thruster channel and near-field plume. [1], [2] The goal is to evaluate the hybrid-DK model through comparison with Koo and Boyd's hybrid-PIC model. [3]
- The kinetic boundary conditions in the DK model, particularly those used to simulate wall interactions, are developed in this work.

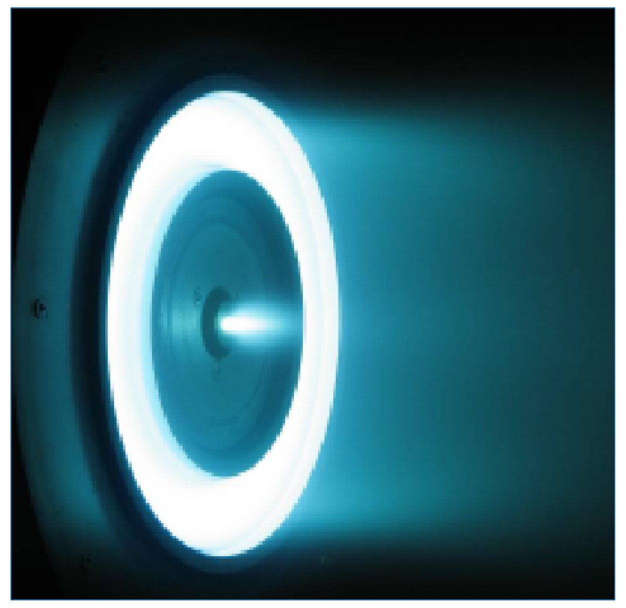

Fig. 1: Xenon Hall Thruster [4]

VDF Overview and Algorithm Flowchart

- The Boltzmann equation describes advection in both physical and velocity space:

$$\frac{\partial f}{\partial t} + \boldsymbol{v}\cdot\frac{\partial f}{\partial z} + \boldsymbol{a}\cdot\frac{\partial f}{\partial v} = S$$

- The number density and mean velocity in a physical cell are described by moments of the VDF:

$$n = \int_{-\infty}^{\infty} f dv_x dv_y = \int_{-\infty}^{\infty} n\cdot f dv_x dv_y$$

$$\bar{u} = \frac{\int_{-\infty}^{\infty} v\cdot f dv_x dv_y}{n}$$

- The following flowchart shows the order of operations involved in updating the VDF for either neutral atoms or ions.

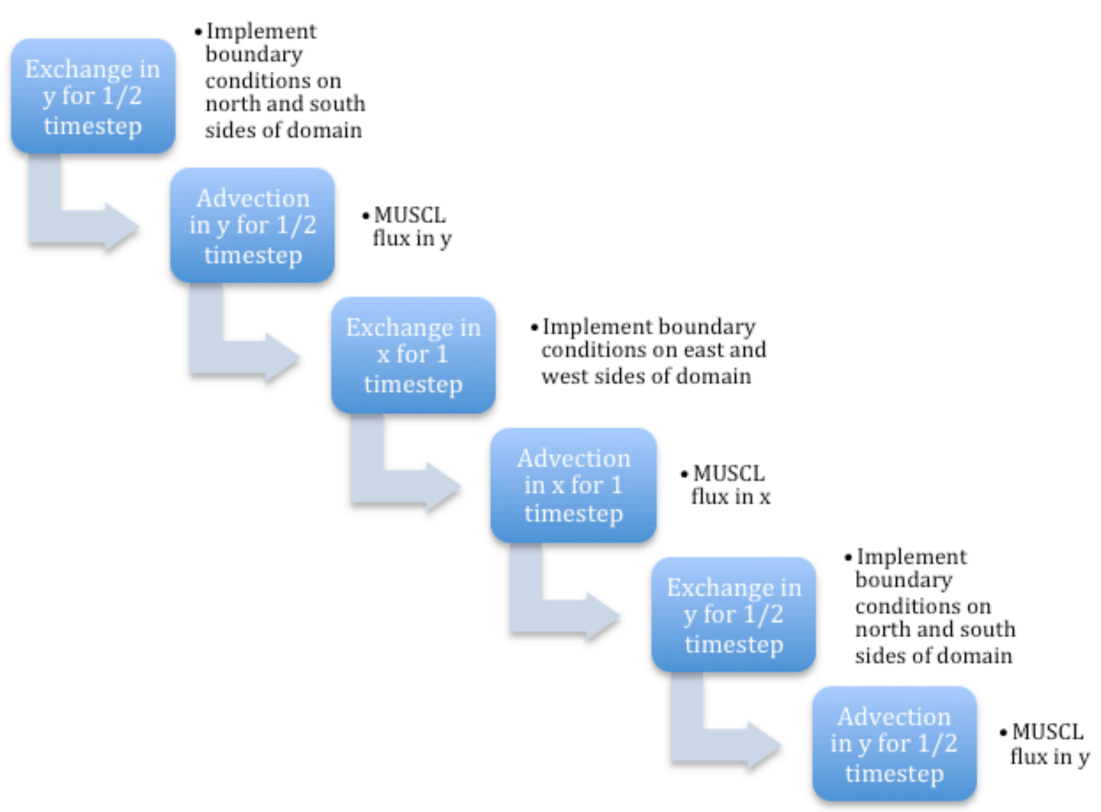

Fig. 2: Strang time splitting for VDF update

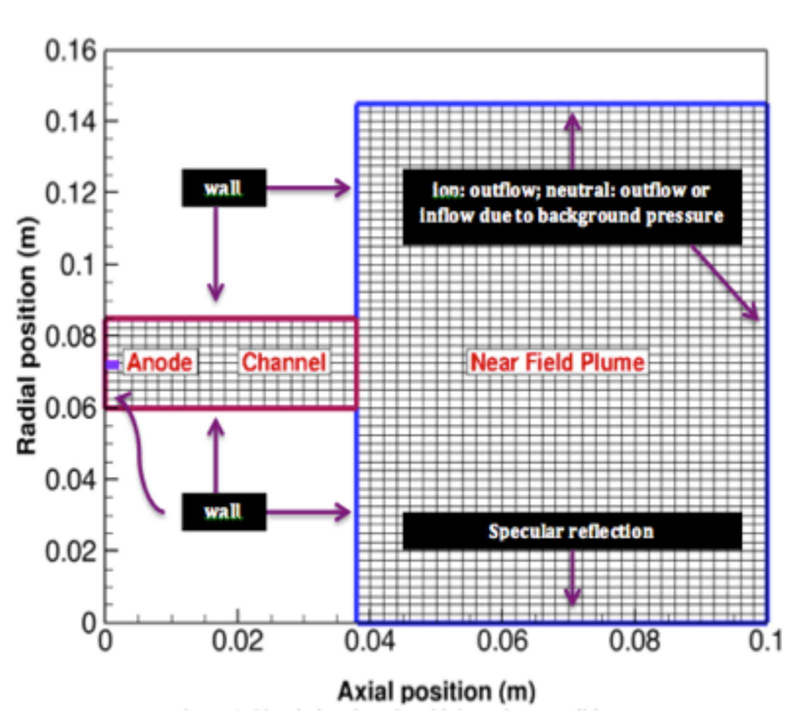

Fig. 3: Simulation domain with boundary conditions

MUSCL Scheme

- MUSCL scheme: monotonic upwind scheme for conservation laws: a finite volume, second order, total variation diminishing (TVD) scheme used to calculate the flux [5], [6]
  - Uses a modified Arora-Roe nonlinear limiter, *Ψ(r)*, which acts to limit numerical extrema including undershoot and overshoot [1]. It is important to preserve positivity.
- Consider one-dimensional linear advection: $\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0$

$$\frac{U_i^{n+1} - U_i^n}{\Delta t} = \frac{-v}{\Delta x}\left(U_{i+\frac{1}{2}}{}^n - U_{i-\frac{1}{2}}{}^n\right)$$

$$U_i^{n+1} = U_i^n - \left(F_{i+1/2} - F_{i-1/2}\right)$$

- The MUSCL scheme becomes useful in estimating *U* (VDF) at the cell centers. For advection with *v* > 0 (positive velocity), the flux through the cell interface can be described by:

$$F_{i-\frac{1}{2}, positive\ CFL} = cU_{i-1} + \frac{(1-|c|)c}{2}(U_i - U_{i-1})\Psi(r_{i-1/2, positive})$$

$$r_{i-\frac{1}{2}, positive\ CFL} = \frac{U_{i-1} - U_{i-2}}{U_i - U_{i-1}}$$

- If the slope factor, *r* ≤ 0.0, then the net flux passing through the interface can only come from the adjacent cell upstream, and *Ψ(r)* = 0. If *r* > 0, the VDF update includes an additional term which utilizes information further upstream, modified by the nonlinear limiter.

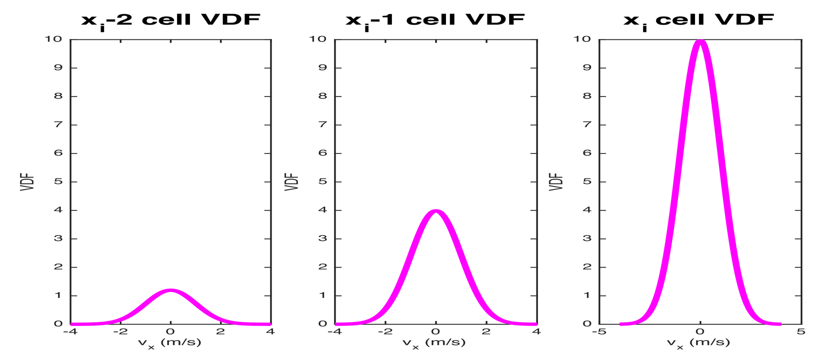

Fig. 4: Ψ(r) ≠ 0 for v > 0 with a positive slope factor

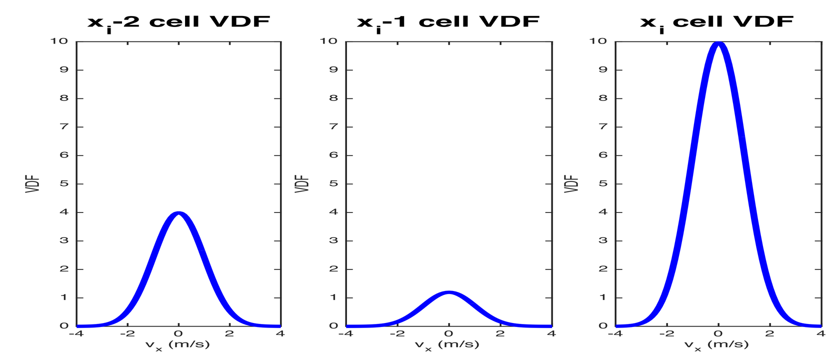

Fig. 5: Ψ(r) = 0 for v > 0 with a negative slope factor

Implementation of Boundary Conditions

- Outflow condition: outflow portion of the VDF (*v* > 0) in the boundary cells is extrapolated to the two adjacent ghost cells, and the other portion is set to zero.

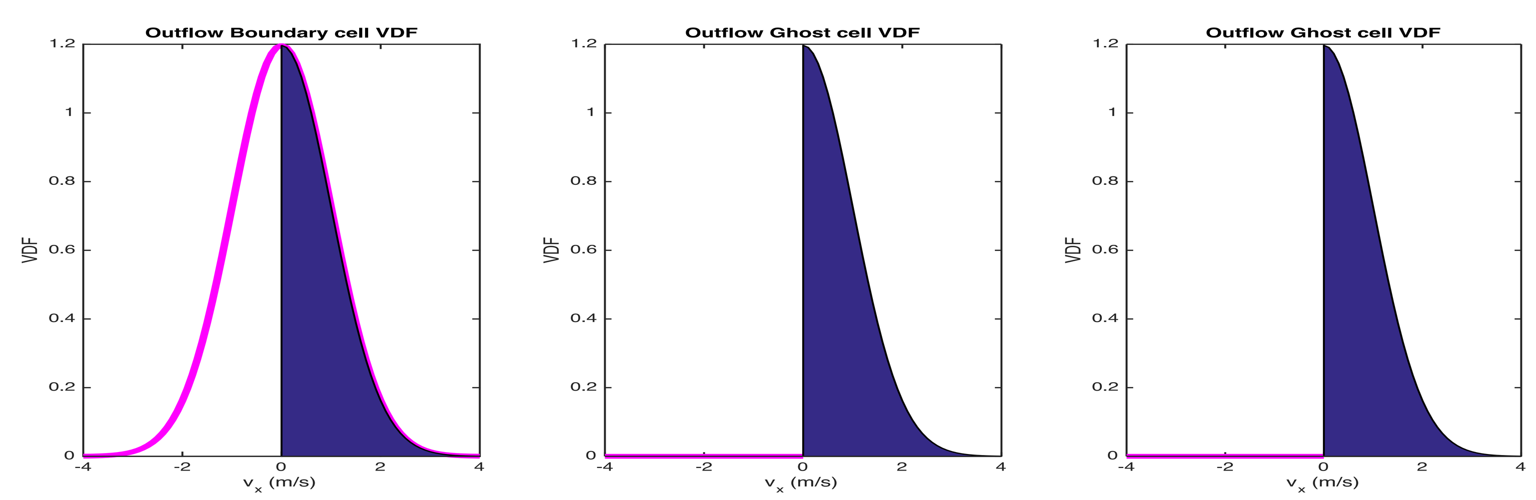

Fig. 6: Sample outflow VDF condition for positive advection

- Inflow due to background pressure: assume that any particles outside the domain will enter with a half Maxwellian distribution at the background temperature.
- Inflow at the thruster centerline: this requires a coordinate transform to mirror the velocity, which is equivalent to a specular reflection in a two-dimensional system. Note that this approach does not take into account the out-of-plane geometry which affects the normal velocity component.
- Reflection at wall boundaries: ions recombine to atoms and reflect diffusely, and neutral atoms simply reflect diffusely.
  - Conduct the ion calculation first. Store the VDF at the wall and calculate the corresponding average ion number density. This is equal to the recombined neutral density, which re-enters the domain as a half Maxwellian based on the chamber temperature. This step is one of the primary challenges, as the total number of particles in the system must be conserved.
  - The same procedure applied for neutral atoms, but the neutral VDF is stored, and the neutral number density is calculated for particle reflection into the domain at the channel wall temperature.

Application to hybrid-DK model

- The following two-dimensional plots show the average ion number density in the simulation domain **before** and **after** changes are implemented. In addition to the kinetic boundary condition updates, ion velocity space is increased, and the minimum threshold density is removed.

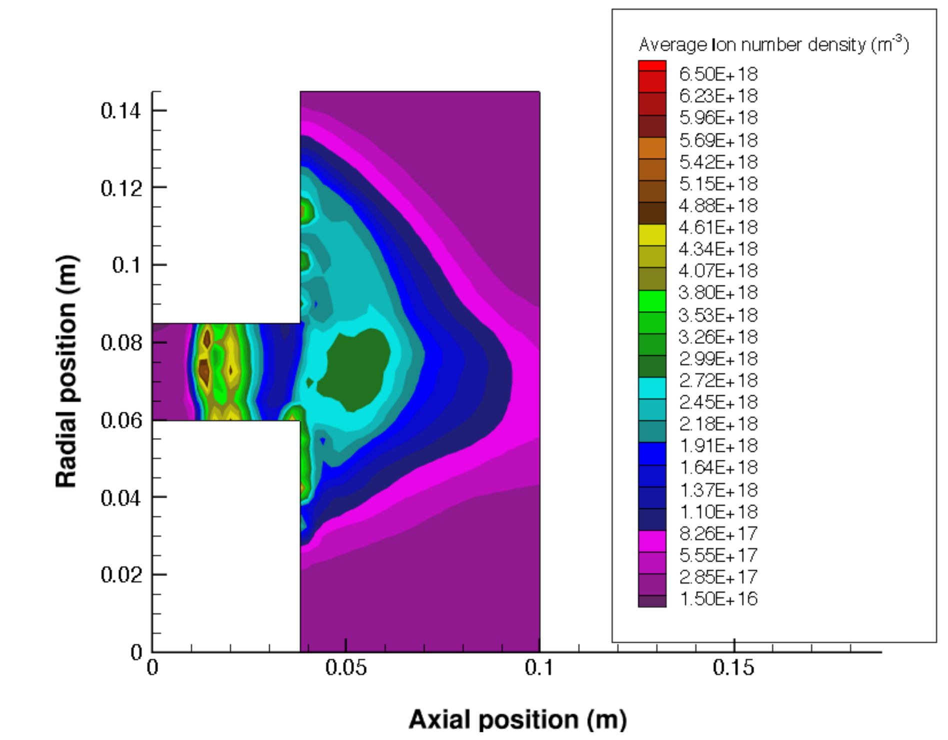

Fig. 7: Average ion density before simulation update

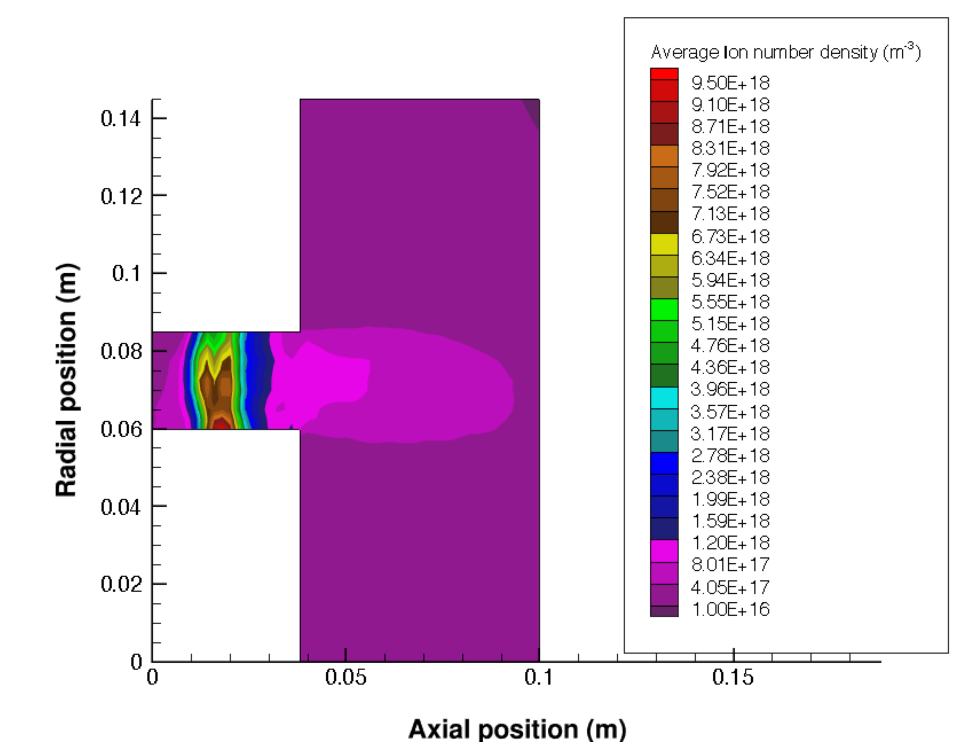

Fig. 8: Average ion density after simulation update

Conclusion

- Kinetic boundary conditions are investigated and updated in the Hall thruster DK simulation domain. Excess ionization is not present in the plume, which indicates that the boundary conditions have been accurately implemented. In the future, the boundary condition at the thruster centerline should be further investigated to account for out-of-plane geometry which affects the normal velocity. This work will be finalized by benchmarking the DK and PIC methods.

Acknowledgements

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References

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