

➤ Background

- Kinetic theory aims to describe gases on a macroscopic level by analyzing their motion on a microscopic level. Particle interaction may be approximated with the hard sphere model, an inverse power law, and others.
- The Boltzmann Equation results, which must be solved in time, space, and velocity, for a given distribution. The right side is the collision term.

$$\frac{\delta}{\delta t}(nf) + C \frac{\delta}{\delta x}(nf) + a \frac{\delta}{\delta C}(nf) = \frac{\delta}{\delta t}(nf)_c$$

➤ Motivation

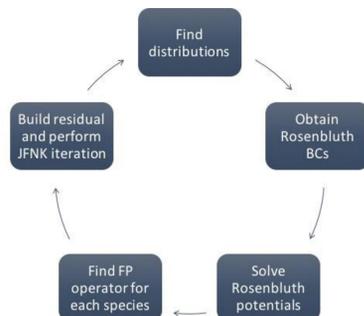
- While often the effect of the collision term is assumed to be negligible in a plasma (the Vlasov equation), systems such as Hall effect thrusters for spacecraft propulsion rely on collisions for their operation.
- The Fokker Planck model, based on an inverse square potential, offers higher accuracy than the widely used BGK model while being more efficiently handled numerically than the explicit collision term.

➤ Methods

- The Rosenbluth-Fokker-Planck operator is given by the following diffusion-advection equation, based on the Rosenbluth potentials:

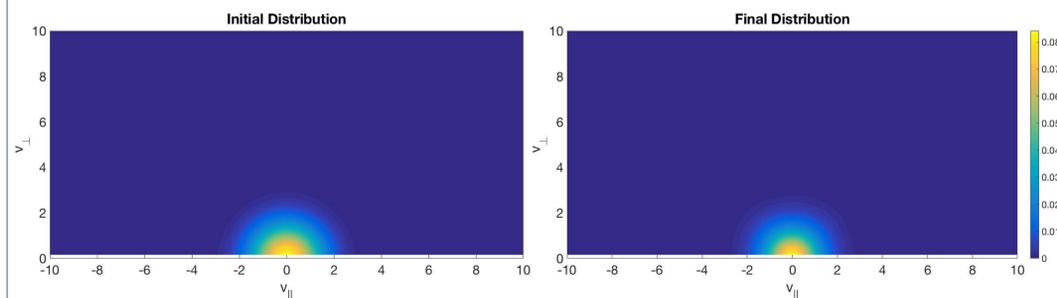
$$\frac{\delta}{\delta t}(f_\alpha)_c = \Gamma_{\alpha\beta} \vec{v}_\beta \cdot \left[\vec{D}_\beta \cdot \nabla_v f - \frac{m_\alpha}{m_\beta} \vec{A}_\beta f_\alpha \right]$$

- The resulting non-linear system is solved through Jacobian-free Newton Krylov (JFNK) methods. This avoids forming or storing the Jacobian matrix.
- The methodology to solve this equation is given by the following chart:

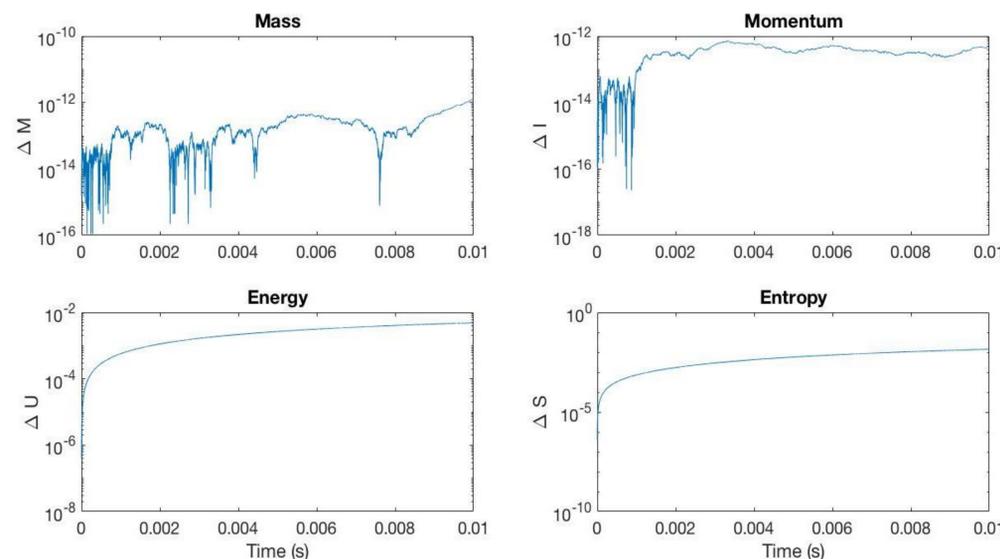


➤ Results

- Simulations were carried out involving a variety of conditions. The most basic case to test the algorithm is to initialize a Maxwellian distribution and ensure this distribution is maintained with time. Given below are the results of such a simulation for an Argon plasma on a 64x32 grid in a cylindrically symmetric domain, using a time step of 1 μs, and run for 10 ms.



- The Maxwellian distribution was maintained with time. Note that the distribution became slightly more peaked; it is shown below that this does not indicate a significant change in the distribution.
- Mass, momentum, energy and entropy were tracked. The change in these properties per time step, as compared to the initial values of the Maxwellian, are given in the following array of plots.



➤ Discussion

- Good conservation of mass, momentum, and energy, in addition to a constant increase in entropy, were achieved by this algorithm; this is crucial in assessing the reliability of future results.
- The simulation grew unstable if the distribution neared the boundaries; better handling of boundary conditions may fix this problem.
- The ability of this algorithm to produce meaningful results was very sensitive to a variety of inputs, such as: time step, mesh refinement, and domain bounds. Making the simulation more robust to setup parameters would greatly reduce time for simulation.

➤ Conclusions

- Future work would benefit from preconditioning of the JFNK iteration and using faster elliptic schemes, such as conjugate gradient methods. In addition, as the current implementation is written in MATLAB, making the program more efficient through vectorizing or parallelization could save significant computation time. Multi-species functionality is also desirable in the future, to describe more complex and physically realistic systems.
- A Rosenbluth-Fokker-Planck algorithm has been implemented here for single-species, showing good conservation of the necessary properties. This allows for an efficiently solved model of collisions in a plasma, yielding results which more closely model physical plasma systems and allowing for the study of relaxation of a plasma towards equilibrium.

➤ Acknowledgements

- Taitano, William T., et al. "A mass, momentum, and energy conserving, fully implicit, scalable algorithm for the multi-dimensional, multi-species Rosenbluth-Fokker-Planck equation." *Journal of Computational Physics* 297 (2015): 357-380.
- This work was generously supported by the Air Force Research Laboratory (AFRL) and the Nonequilibrium Gas and Plasma Dynamics Laboratory (NGPDL) at the University of Michigan.
- Distribution A: Approved for public release; unlimited distribution, PA# 17606.