An Exact Hot-Tube Solution for Thin Tape Helix Traveling-Wave Tube*

Patrick Y. Wong¹, David P. Chernin², Y. Y. Lau¹, Ronald M. Gilgenbach¹, and Brad W. Hoff³

¹University of Michigan, Dept. of Nuclear Engineering and Radiological Sciences, Ann Arbor, MI
²Leidos Inc., Reston, VA
³Air Force Research Laboratory, Kirtland, NM

8th MIPSE Graduate Student Symposium
18 October 2017

*Work supported by AFOSR Grant No. FA9550-15-1-0097.
Motivation

• Traveling-Wave Tubes (TWTs) are amplifiers used in satellite communications

• Gain is governed by Pierce dispersion relation for the beam-circuit interaction:

\[
[(\beta - \beta_e)^2 - 4\beta_e^2 QC^3][\beta_{ph} - \beta] = \beta_e^3 C^3
\]

- **Beam Mode**
- **Circuit Mode**
- **Coupling**

\[\beta = \text{propagation constant}\]
\[\beta_e = \omega/v_0\]
\[\beta_{ph} = \omega/v_{ph}\]

\[C = \text{gain parameter}\]
\[Q = \text{space-charge parameter, very difficult to determine}\]

**This work:** Determine \(Q\) reliably for first time from an exact theory.
Exact Solution for Tape Helix TWT

- Cold-tube (no beam) dispersion relation derived exactly in [1]

- Add in a pencil beam. Obtain exact hot-tube dispersion relation

Previous Models of $Q$ (or $QC$)

- **Branch & Mihran [2]:**
  - Replaces the helix with a perfectly-conducting tube (commonly used)

- **Sheath Helix [3]:**
  - Improvement on Branch & Mihran: current follows helical path
  - Used in the large-signal helix TWT codes

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The Hot-Tube Dispersion Relation

\[ D(\beta; \omega) = \det(M) = 0, \]

\[ M_{ll'} = (-1)^l j^{l+l'} \sum_{n=-\infty}^{\infty} \begin{pmatrix} J_l(\alpha_n) & 0 \\ 0 & \frac{l+1}{\alpha_n} J_{l+1}(\alpha_n) \end{pmatrix} \overline{Z_n} \begin{pmatrix} J_{l'}(\alpha_n) & 0 \\ 0 & \frac{l'+1}{\alpha_n} J_{l'+1}(\alpha_n) \end{pmatrix}, l, l' = 0, 1, 2, \ldots \]

If beam current \( \rightarrow 0 \), have verified analytically that the cold-tube dispersion relation is recovered.
Cold-tube limit (numerical)
Test Case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tape radius ((a))</td>
<td>0.1245 cm</td>
</tr>
<tr>
<td>Pitch ((p))</td>
<td>0.0801 cm</td>
</tr>
<tr>
<td>Helix pitch angle ((\psi))</td>
<td>5.85°</td>
</tr>
<tr>
<td>Wall radius ((b))</td>
<td>0.2794 cm</td>
</tr>
<tr>
<td>Dielectric constant of supporting layer ((\varepsilon_r^{(2)}))</td>
<td>1.25</td>
</tr>
</tbody>
</table>

With \(\frac{w}{pcos(\psi)} = 0.2\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam radius ((r_b))</td>
<td>0.05 cm</td>
</tr>
<tr>
<td>Beam voltage ((V_b))</td>
<td>3 kV</td>
</tr>
<tr>
<td>Beam current ((I_0))</td>
<td>0.17 A</td>
</tr>
</tbody>
</table>
Hot-Tube Results
Fixed Beam Current $I_0$

Always numerically found three roots, $\beta_1$, $\beta_2$, $\beta_3$, with two of the roots occurring in complex conjugates.
Hot-Tube Results
Fixed Signal Frequency (4.5 GHz)
A New Space-Charge Parameter, \( q \)

- Impossible to construct the Pierce parameters \( Q \) and \( C \) self-consistently from the three exact roots, \( \beta_1, \beta_2, \beta_3 \):
  \[
  (\beta - \beta_1)(\beta - \beta_2)(\beta - \beta_3) = 0
  \]
- Need to introduce an additional space-charge parameter, \( q \)
  \[
  \left[ (\beta - \beta_e)^2 - 4\beta_e^2 Q C^3 \right]\left[ (\beta_{ph} - \beta) + 4\beta_{ph} q C^3 \right] = \beta_e^3 C^3
  \]

\( Q \): Modification of beam mode by space-charge effects
\( q \): Modification of circuit mode by space-charge effects

**Note:** \( q \) is like a detune in the circuit, so it could be very important.
Determination of $Q$, $q$, and $C$

from $\beta_1, \beta_2, \beta_3$

\[
\begin{align*}
qC^3 &= \frac{1}{4} \left( \frac{\beta_1 + \beta_2 + \beta_3 - 2\beta_e}{\beta_{ph}} - 1 \right) \\
QC^3 &= \frac{1}{4} \left( 1 - \frac{\beta_1 \beta_2 + \beta_2 \beta_3 + \beta_1 \beta_3 - 2\beta_e \beta_{ph} (1 + 4qC^3)}{\beta_e^2} \right) \\
C^3 &= \frac{\beta_e^2 (1 - 4QC^3) \beta_{ph} (1 + 4qC^3) - \beta_1 \beta_2 \beta_3}{\beta_e^3}
\end{align*}
\]
The Pierce Parameters
Fixed Beam Current $I_0$
The Pierce Parameters
Fixed Signal Frequency (4.5 GHz)
Conclusions

• An *exact* hot-tube dispersion relation for a realistic tape helix TWT was derived, for the first time.

• **Discovered a new parameter** $q$ **that accounts for space-charge effects on circuit mode**, possibly as important as the well-known $Q$ that accounts for space-charge effects on beam mode.

• $Q$, $q$, and $C$ are extracted from the exact dispersion relation, necessary for non-linear code development.