

B. Virial Theorem & Biot Savart Law

Ring Current as a collection of particles

- Close to earth the dipole field acts as a magnetic mirror with strong gradients and curvature which traps particles.
- Particle motions determined by kinetic energy, sum over all particles to obtain total ring current energy W_{RC} can be used to obtain ΔB

$$\Delta B_{RC} = \frac{-\mu_0}{2\pi} \frac{W_{RC}}{B_E R_E^3} \hat{e}_{dipole}$$

Virial Theorem

- Start with ideal MHD momentum $\frac{\partial(\rho \mathbf{V})}{\partial t} + \nabla \cdot \left(\rho \mathbf{V} \mathbf{V} + p \mathbf{I} + \frac{B^2}{2\mu_0} \mathbf{I} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} \right) = 0$
- Virial: $\int d\mathbf{r} [\text{momentum}] \cdot \mathbf{r} = 0$
- Subtract virial of empty dipole, collect terms and simplify [1]
- Relates magnetic perturbation, surface momentum flux (forces), and volume integrated energies

$$\mu \cdot \mathbf{b}(0) = - \int dS \cdot \mathbf{r} \left[p_{th} + \frac{B^2}{2\mu_0} \right] - \int dS \cdot \left[\frac{\partial}{\partial t} (\rho \mathbf{V}) \frac{r^2}{2} + \rho \mathbf{V} (\mathbf{V} \cdot \mathbf{r}) \right] + \int dS \cdot \frac{\mathbf{B}}{\mu_0} [\mathbf{B} - \mathbf{B}_d/2] \cdot \mathbf{r} + \int dS \cdot \frac{1}{2\mu_0} [\mathbf{B}_d \times (\mathbf{r} \times \mathbf{B}_d)] + 2U_k + U_b + U_G$$

Biot Savart law

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{(\mathbf{J} dV) \times \mathbf{r}'}{|\mathbf{r}'|^3}$$

- Directly calculate perturbation by integrating current densities
- Indirectly related to energy

C. Magnetosphere System

Magnetosphere Definition

- Dominated by planetary dipole field
- Pressure balance with shocked solar wind

Magnetic Reconnection

- Ideal MHD plasma B is "frozen in" magnetic flux is transported with the flow
- Reconfiguration of magnetic topology to lower magnetic potential energy.
- Converts to thermal and kinetic energy.

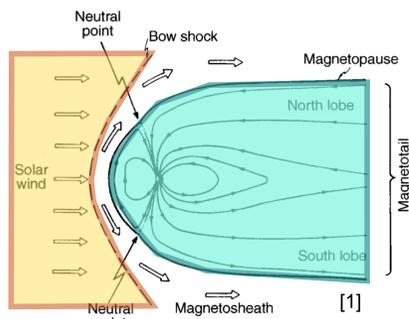
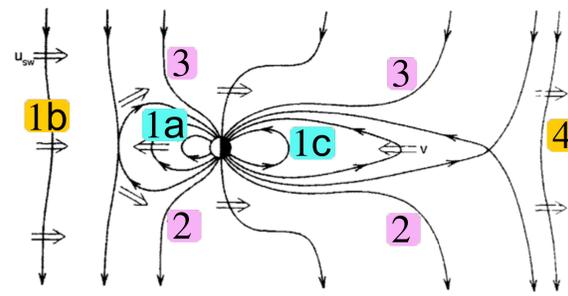


Fig. 2: Diagram of the solar wind and magnetosphere. [1]

Dungey Cycle



- Dayside magnetosphere (1a) and incoming solar wind (1b) undergo magnetic reconnection creating open flux (2 & 3).
- Newly open flux (2 & 3) is advected with the surrounding flow and reconnects again in the tail leading to closed (1c) and solar wind (4) configurations
- Favorable when solar wind magnetic field $B_z < 0$

Fig. 3: Diagram of Dungey cycle. [2]

A. Plasma in Geospace

Background: Space plasma affects life on earth

- Plasma in **geospace** affects our lives on the ground.
- Induced currents are generated by rapidly changing perturbations to the planetary dipole magnetic field.
- Using simulation, magnetic perturbation can be calculated directly or related to **energy** state and surface properties of an arbitrary volume via the **Virial Theorem**.
- Energy changes in specific regions of the system can be linked to external an internal dynamics
- Virial theorem allows quantification of specific dynamics to observed magnetic perturbations

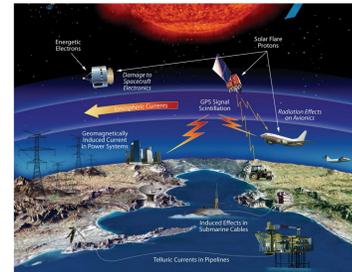


Fig. 1: Sensitive infrastructure can be damaged by magnetic perturbations.

F. Contributions to ΔB

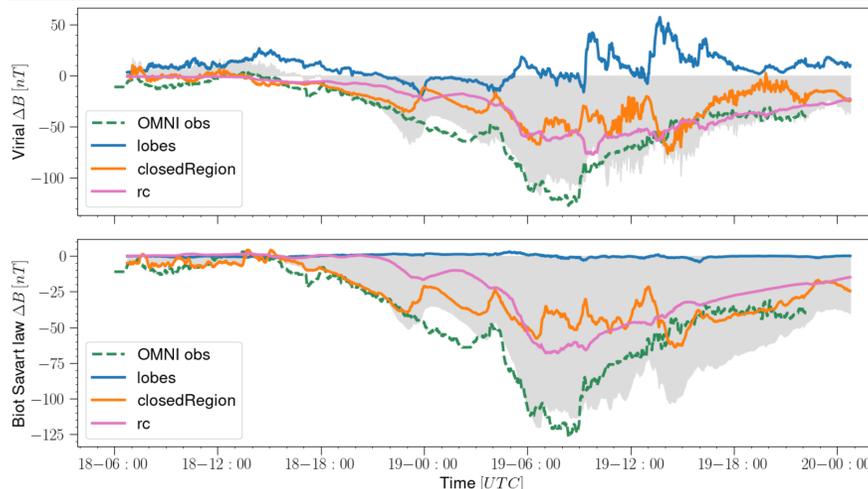


Fig. 6: Time traces showing magnetosphere sub-volume contributions to magnetic perturbation according to Virial and Biot Savart law formulations.

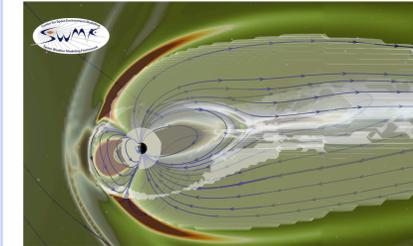
- Overall prediction of magnetic perturbation for Virial formulation, Biot Savart law formulation, and observations agree.
- Surface terms of virial oppose volume contributions from sub-volumes, combined with Lobes and Closed region based on volume weighted average.
- Virial formulation predicts significant contribution from lobe magnetic perturbation energy, will be offset by surface terms.
- Primary contributions from ring current as expected and from Closed region.

G. Conclusion

- Virial Theorem can accurately predict magnetic perturbation and agrees with Biot Savart law calculation.
- Evidence of magnetic reconnection observed by comparing the distribution of energy between magnetospheric regions. Future work to separate and quantify external energy flux, dayside reconnection, and tail reconnection effects.

D. Methodology

Space Weather Modeling Framework



Region	Description
Global Magnetosphere	Ideal MHD
Inner Magnetosphere	Kinetic drift
Ionosphere	Electrodynamics

Real Event Simulation

- Complex storm event
- 3D data output at 1min intervals
- Sub-volumes identified at each output

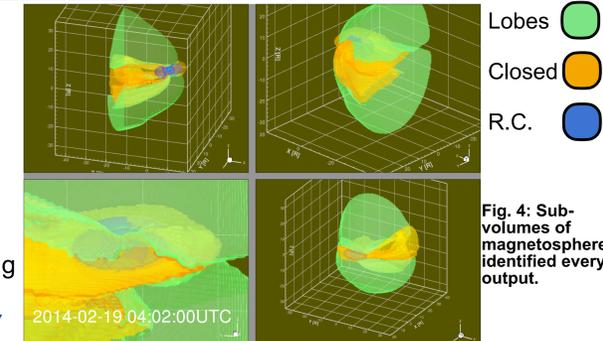


Fig. 4: Sub-volumes of magnetosphere identified every output.

Region Selection Criteria

- Magnetic field line tracing
- Dipole coordinate geometry

E. Energy Partitions

- $t_1 \rightarrow t_2$: Period of southward solar wind magnetic field favorable to reconnection shows steady increase in Lobe magnetic energy.
- As Lobe energy increases, the Closed region also increases, consistent with the Dungey cycle. Its total energy is made up of both magnetic and thermal.
- t_2 : As solar wind magnetic field turns northward and dynamic pressure increase the energy flow to the lobes is cutoff but energy in the Closed region rapidly increases as Lobe energy is depleted.

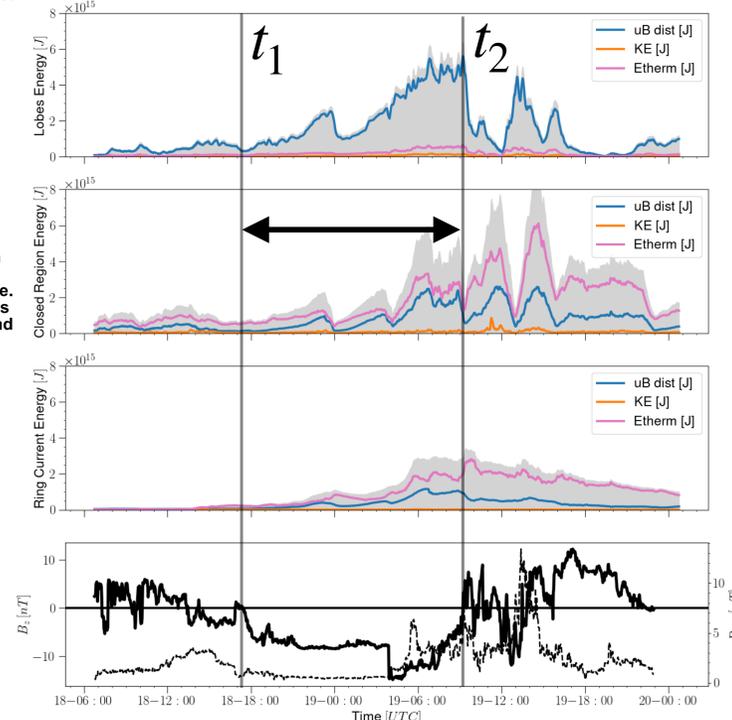


Fig. 5: Time traces of integrated energy in each sub-volume of magnetosphere. Grey shading is total energy and curves are energy types.

REFERENCES

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- Lyons, L. R., and Williams, D. J., *Quantitative Aspects of Magnetospheric Physics*, 1984.

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