Grating Optimization for Smith-Purcell Radiation: Direct Correlation between Spatial Growth Rate and Starting Current

Md Arifuzzaman Faisal, Peng Zhang
Department of Electrical and Computer Engineering, Michigan State University

Email: {faisalmd, pz}@egr.msu.edu

TABLE I. Main Parameters for the Calculation.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam energy</td>
<td>50 keV</td>
</tr>
<tr>
<td>Beam Height from Grating Surface, a</td>
<td>10 μm</td>
</tr>
<tr>
<td>Beam thickness, t</td>
<td>10 μm</td>
</tr>
<tr>
<td>Beam Current, I</td>
<td>5000 A/m</td>
</tr>
<tr>
<td>Grating period, L</td>
<td>120 μm</td>
</tr>
<tr>
<td>Number of grating periods</td>
<td>35</td>
</tr>
</tbody>
</table>

I. Motivation

- Back wave oscillation (BWO) based Smith-Purcell Radiation (SPR) has attracted strong interests for producing Terahertz (THz) radiation.
- High-power, efficient, and low-cost electromagnetic sources have significant uses in high-resolution imaging, biomedical scanning, material analysis, security systems, and high-rate data communications (6G), etc.

II. Smith-Purcell Radiation

The cold-tube dispersion relation for SPR is [2,3],

\[ \frac{\text{cot}(\beta L)}{\beta L} - \sum_{n=-\infty}^{\infty} \frac{\sin(\beta n L)}{n^2 \beta n L} = 0 \]  

(1)

where the normalized grating width \( W = w/L \), grating height \( H = h/L \), frequency \( \omega = \omega_0/c \), wavenumber \( k = k_0 \), \( \beta_0 = \left( \frac{\omega_0}{c} \right) \sqrt{\frac{\omega_0}{\epsilon}} \), and \( k_0 = k + 2\pi n \).

The cold-tube dispersion relation \( \omega(k) \) (Eqn. 2) for different groove’s height \( h \) and width \( w \). The yellow dots denote the operation points at the evanescent wave frequency \( \omega_{ev} \) [4].

III. Cold-Tube Dispersion Relation

The cold-tube dispersion equation for the SPR model is [2,3],

\[ \frac{\text{cot}(\beta L)}{\beta L} - \sum_{n=-\infty}^{\infty} \frac{\sin(\beta n L)}{n^2 \beta n L} = 0 \]  

(2)

The hot-tube dispersion relation for the SPR model is [4],

\[ f(k; \omega) \equiv \cot(\beta L) - \sum_{n=-\infty}^{\infty} \frac{\sin(\beta n L)}{n^2 \beta n L} \frac{\omega_0}{\omega} = 0 \]  

(3)

where the normalized beam diameter from the grating \( A = a/L \), \( \Omega = \omega_0 \sqrt{\frac{\omega_0}{\epsilon}} - \frac{p_b}{\omega_0} \), \( \alpha_0 = \omega_0 \sqrt{\frac{\omega_0}{\epsilon}} \), \( \omega_0 = \omega_0/c \), \( n \) is the number of electrons per unit volume, \( \tau \) is the thickness of beam.

The hot-tube dispersion relation (Eqn. 3) is solved numerically for obtaining the complex wavenumber \( k = k_e + jk_i \) for a given operating frequency \( \omega \) with \( \text{exp}(\pi \beta L) \). For deriving Eqn. (3), a negative imaginary part of the wavenumber \( k = k_e + jk_i \) indicates a growing wave as a function of position as \( \exp(k_i x) \), with \( -k_e \) being the growth rate.

IV. Operating Frequency

IV. Operating Frequency

V. Starting Current from PIC Simulation

(a) The operating frequency \( f_{op} \) and (b) the corresponding evanescent wavelength \( \lambda_{ev} \) as a function of grating groove’s height [red line] and width [blue line] [4]. The surface plot of (c) \( f_{op} \) and (d) \( \lambda_{ev} \) for a wide range of grating groove’s heights and widths.

VI. Effect of Grating Parameters

To get accurate starting current for Smith-Purcell Radiation, small step size for increasing current is needed.

VII. Starting Current from PIC Simulation

This figure shows the roots of the Eqn. (3) where the grating parameters are \( a = 60 \mu m \) and \( h = 40 \mu m \) and the corresponding operating frequency \( \omega = 1.8444 \) (Calculated from Eqn. (2)).

VIII. Hot-Tube Dispersion Relation

The above figure shows the (a) real and (b) imaginary part of the wavenumber calculated from hot-tube dispersion relation (Eqn. 3) at the operating frequency \( \omega = 2\pi f_{op} L/c \) calculated from Eqn. (2), as a function of the groove height \( h \) (width \( w \) fixed at 60 \( \mu m \)) as well as groove width \( w \) (height \( h \) fixed at 100 \( \mu m \)), where \( \text{exp}(\pi \beta L) \) (Fig. (b)) represents the growth rate.

IX. Growth Rate

Advantages

We can avoid PIC simulation

1. For finding the starting current we can use above plots and avoid costly Particle-In-Cell simulation.

Computational cost is very low

As we need to solve only cold-tube and hot-tube dispersion relation, Computational cost is very low.

X. Conclusion

The operating frequencies of SPR have changed with different grating parameters using cold-tube dispersion relations where the grating period has been fixed. The growth rate calculation using hot-tube dispersion relation can be used to predict the optimal grating parameters to minimize the starting current of SPR. This approach has a significantly reduced computation cost compared to direct PIC simulations. As both the operation frequency of SPR and its growth rate depend strongly on the grating parameters, both the cold-tube and hot-tube dispersion relations can be used in combination to minimize the starting current at a desired radiation frequency. While we apply our analysis to the effects of grating parameters on SPR, we expect that our dispersion relation treatment to grating optimization is applicable to study linear free-electron beam-based vacuum devices in general and in various geometries (e.g., cylindrical geometry).

References