**Problem:** Hall thruster simulations are not predictive due to incomplete understanding of electron transport physics.

Hall thrusters are electric spacecraft propulsion devices which use crossed electric and magnetic fields to generate and accelerate a plasma to produce thrust. They are widely used for satellite station-keeping, but there is increasing interest in deep space and crewed missions. We would like to use simulation to aid the design of new thrusters. However, current simulations are not predictive!

**Correlation between empirical and self-consistent models of Hall thruster anomalous transport**

**Question:** How can we calibrate and validate proposed models of anomalous electron transport in Hall thrusters?

**Empirically inferring electron mobility**

In the absence of a suitable model for the anomalous mobility, researchers typically make simulations match experiment by prescribing it along the channel centerline of the Hall thruster and varying the spatial dependence of the transport iteratively until the simulations match experiment.

The mobilities inferred from this empirical calibration procedure are often treated as surrogate measurements of the true anomalous mobility in the thruster. 

**Idea:** calibrate self-consistent models by comparing to empirical mobilities

**Generating an empirical reference simulation**

- Calibrated a Hall2De (2D axisymmetric fluid code from JPL) simulation to match time-averaged experimental discharge current and ion velocity profile for H9 Hall thruster at 300V and 15 A.
- Required 29 iterations, each simulation took ~1 day.
- We can match experiment well but thruster is somewhat low.

**Performance metrics**

- Case: 292.0 ± 2.3 mN
- Ion energy: 15 A ± 4.0 %
- Ref. sim: 258.3 mN
- Ion energy: 15.2 A ± 4.9 %

**Comparing empirical and self-consistent models**

- **Models**
  - Two algebraic models investigated. Both integrated into Hall2De and allowed to update at each solver timestep along.
  - First-Principles model derived from assumptions about the scaling of Hall thruster turbulence.
  - Data-Driven model obtained by regressing a dataset of empirical mobilities (not including H9).

- Before simulation, models tuned to match empirical mobility
- After simulation, models have diverged from empirical mobility
- Performance is poor, despite initial agreement
  - First-principles model agrees better with reference simulation despite worse initial agreement
  - Data-driven model matches efficiency better but predicts discharge current 2x experiment
  - Velocity profile of data-driven profile much more shallow than empirical profile due to high mobility in the acceleration region.

- **Takeaway:** agreement with empirical mobility does not guarantee model quality. Why does this occur? What does it mean?

**Discussion**

**Reasons for divergence from empirically-inferred mobility**

1. Non-linearity in governing equations means small deviations from empirical mobility can be amplified.
2. Hall thrusters are oscillatory, so evaluating models on time-averaged data is subject to artifacts (product of averages is not equal to the average of products).
3. Empirically-inferred mobilities are non-unique and large changes in mobility in certain parts of the device may not change observables much.

**Implications for modeling anomalous transport**

Instead of inferring static anomalous mobility using simulations, should try to measure it experimentally as a function of time. This would give much more information about the dynamic behavior of the transport and would provide more data for model calibration and validation.

**Conclusion**

Empirically-inferred mobility profiles are useful for making simulations match experiment. However, they should not be treated as surrogate measurements of the anomalous transport. Models should instead be compared to direct time-resolved measurements of the anomalous mobility (if available) or implemented directly into simulations to gauge their performance.

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