In Figure 1, we demonstrate that after equilibration N-S equations are a reduced model of Grad’s moment equations. In Figure 2, we plot Rowes and Saul’s inverse reconstruction error (PCA) ability to reduce the ensemble’s dimension. We demonstrate that the data can reduce from 10 (5 complex values) to 6 dimensions at late times; which matches the reduction from Grad’s moments ($\rho_k, u_k, T_k, \sigma_k, q_k$ each is a complex value) to N-S moments ($\rho_k, u_k, T_k$).

In Hartman-Groebn theory, the Jacobian of a non-linear system is used near an equilibrium to linearize. As an improvement, the Koopman operator can be used away from equilibrium to linearize the dynamics. Like the Jacobian, the eigensystem of the Koopman operator characterizes the slow manifold [6]. DMD is a known way to discover a finite dimensional representation of the Koopman operator, $K$ and its eigenvalues $\Lambda$ and vectors $\Psi$. We conduct DMD on a window of the data (i.e., subsection of the sequential data represented as a blue band in Figure 3: left). Windowed DMD produces a timeseries of eigenvalues, see in Figure 3: right. We observe that two of the eigenvalues have an absolute value that differs by many orders of magnitude by $t = 0.5$, which is a similar time scale as seen in the dimension reduction plots. Furthermore, plots of the similarity between the DMD eigenvectors at time $t_1$ and $t_2$ are presented in Figure 4. We observe that after $t = 0.5$, DMD rediscovers the same eigenvector; this indicates that the slow manifold has been identified.