

# Universal Scaling of the Electron Distribution Function in Relativistic Laser-Solid Interactions

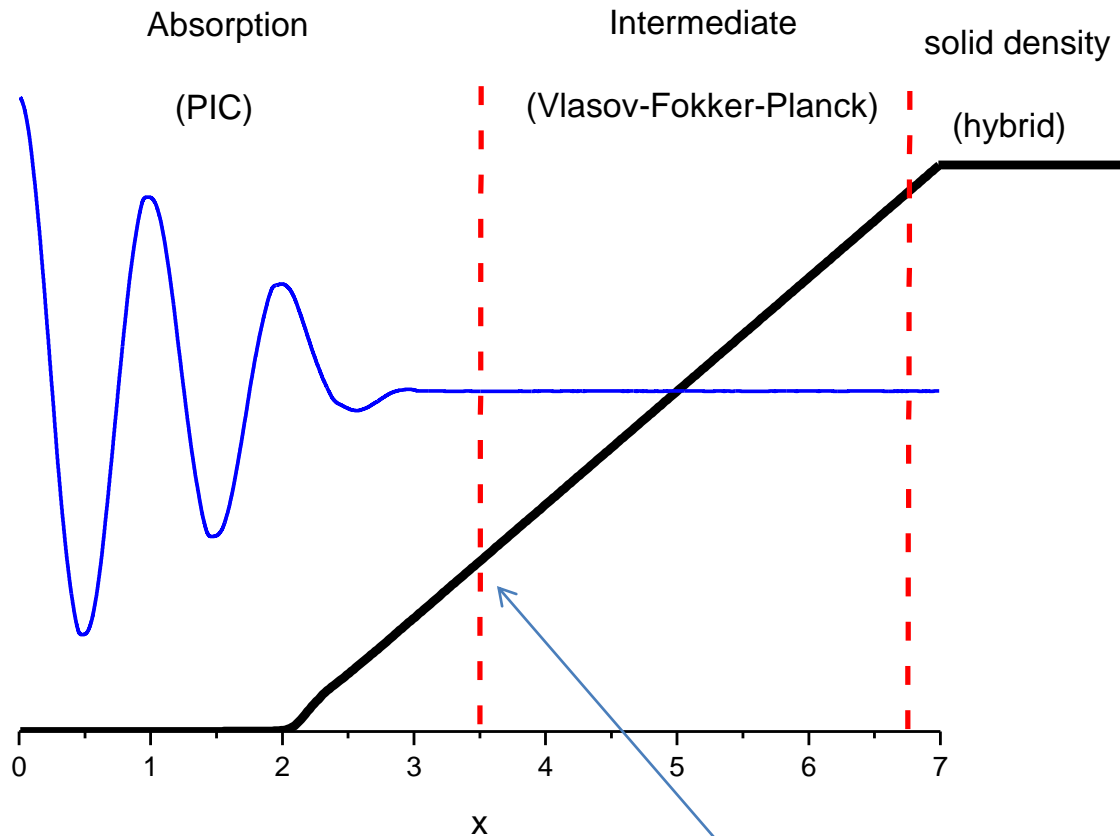
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Acknowledgements

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# The Intermediate Regime



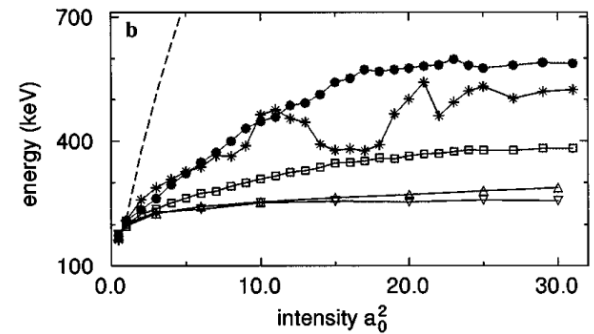
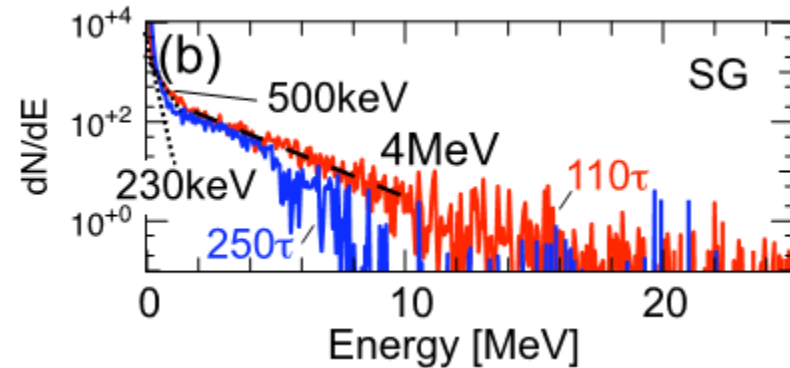
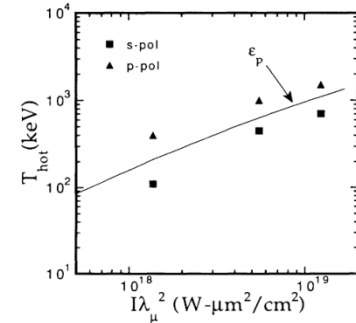
Source of fast electrons?

# Basic Questions in Short-Pulse Fast Ignition

- What are the fast electron characteristics: **energy, momentum, density, energy spread, angular spread, current** etc.? In other words can we come up with a simple formula for  $f(\mathbf{p})$ ? This hasn't even been done in 1D.
- In general the characteristics of the fast electrons is sensitive to how you decide to define what is “fast”. Can we find a method that is relatively insensitive? (i.e. a relatively objective definition)

# Previous Studies

- Wilks 1992: “heated electron temperature versus intensity”. How? Where? When? **Ponderomotive scaling.**
- Kemp 2008 : Assumed Maxwellian and fitted a line to the  $dN/dE$  spectrum. Result was independent of where measured.  $I=1.37 \times 10^{20} \text{Wcm}^{-2}$ . Also Pukhov, Patel.
- Lefebvre & Bonnaud 1997: take average energy  $>100\text{keV}$ . Time and space averaged. **Much lower than ponderomotive.**



Several people have looked at the issue of FE energy scal. most not. naravno Wilks who showed pondero. scal. ali nazalost za nas didn't mention kako, gde i kad the FE energy was calc'd. Some people (kao Kemp i Phukhov) have attempt. to def. the FE temp. by assum. a Maxw. dist. and fitting a straight line to work back to the average energy. There always seems to be some level of uncertainty as to where to fit the straight line. In contrast to that technq. Lefebvre & B looked at the time and space averaged av. energy. of electrons above 100keV. That produced a result much lower than pond.

# The Model (FIDO)

- Vlasov-Fokker-Planck Equation
- Maxwell's Equations
- Cartesian grid in configuration space
- Spherical grid in momentum space (makes collision algorithms fast and simple, naturally allows for the effect of magnetic fields and  $p$  is natural coordinate to stretch if you want to study two populations)
- Piecewise-Parabolic Interpolation for advection
- Solve for collisions and fields implicitly

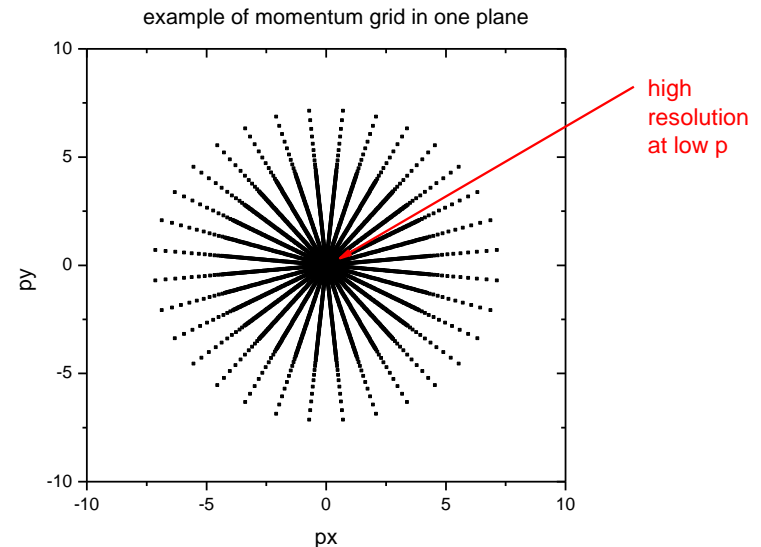
$$\frac{\partial f}{\partial t} + \nabla_r \cdot (\mathbf{v}f) + \nabla_p \cdot (\mathbf{E}f) = C_{ei} + C_{ee}$$

## Results shown today

- number of points in  $p=110$
- number of points in angle=32-96
- 500 cells in  $x$

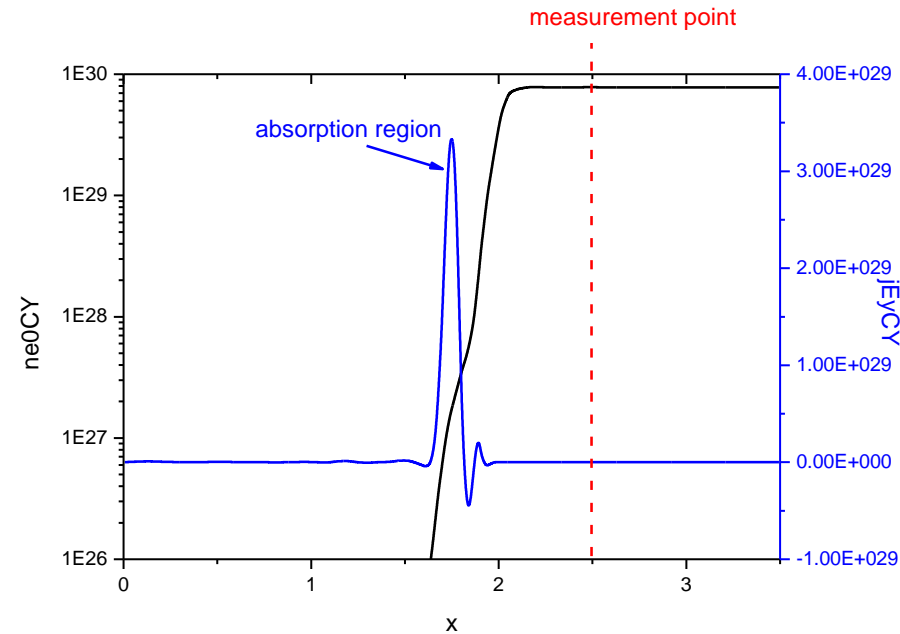
## Simplicity

- Normal incidence
- Steep density profile
- Immobile ions



# Criteria

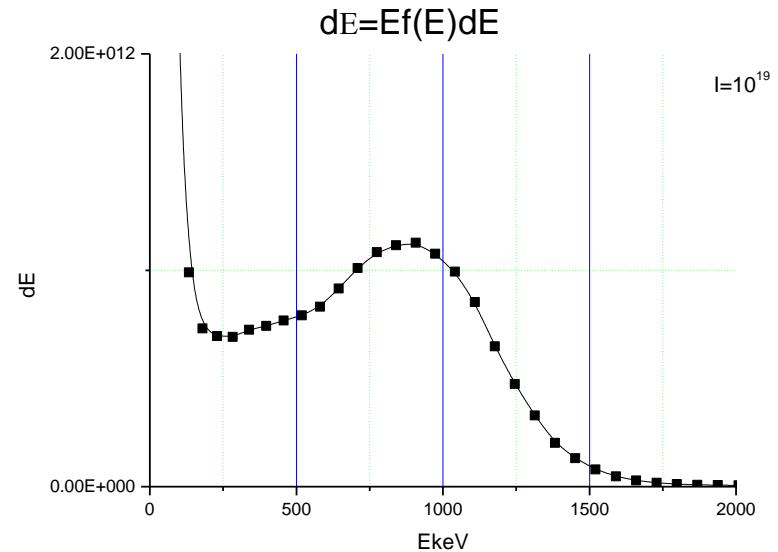
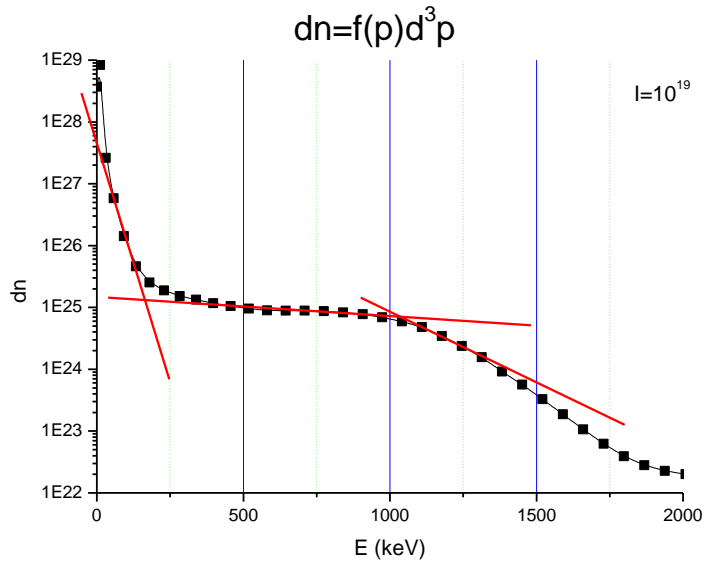
- Where? Just behind absorption region.
- When? After the flux has reached “equilibrium”. Cycle-averaged.
- How? Choose quantities related to those electrons which carry 90% of the heat flux.
- Calculate  $\langle E \rangle$ ,  $E_{\text{peak}}$  and  $E_{\text{spread}}$ .



Sleduci we set the criteria for measuring the dist. func. Ovde na pravo je plot telling us where the laser energy is being absorbed (the blue line) with the black line the density profile. We're going to measure the FE dist. func. at the red line which is about a micron behind the absorption region. When? We give the system plenty of time to reach a cycle-averaged eqmb which corresponds to 60fs. How? Let's only sample those FE's which carry 90% of the heat flux

# $I=1 \times 10^{19} \text{ W cm}^{-2}$ Angle-Averaged $f(E)$

- $dn$  has many “temperatures”



This is the most common approach.

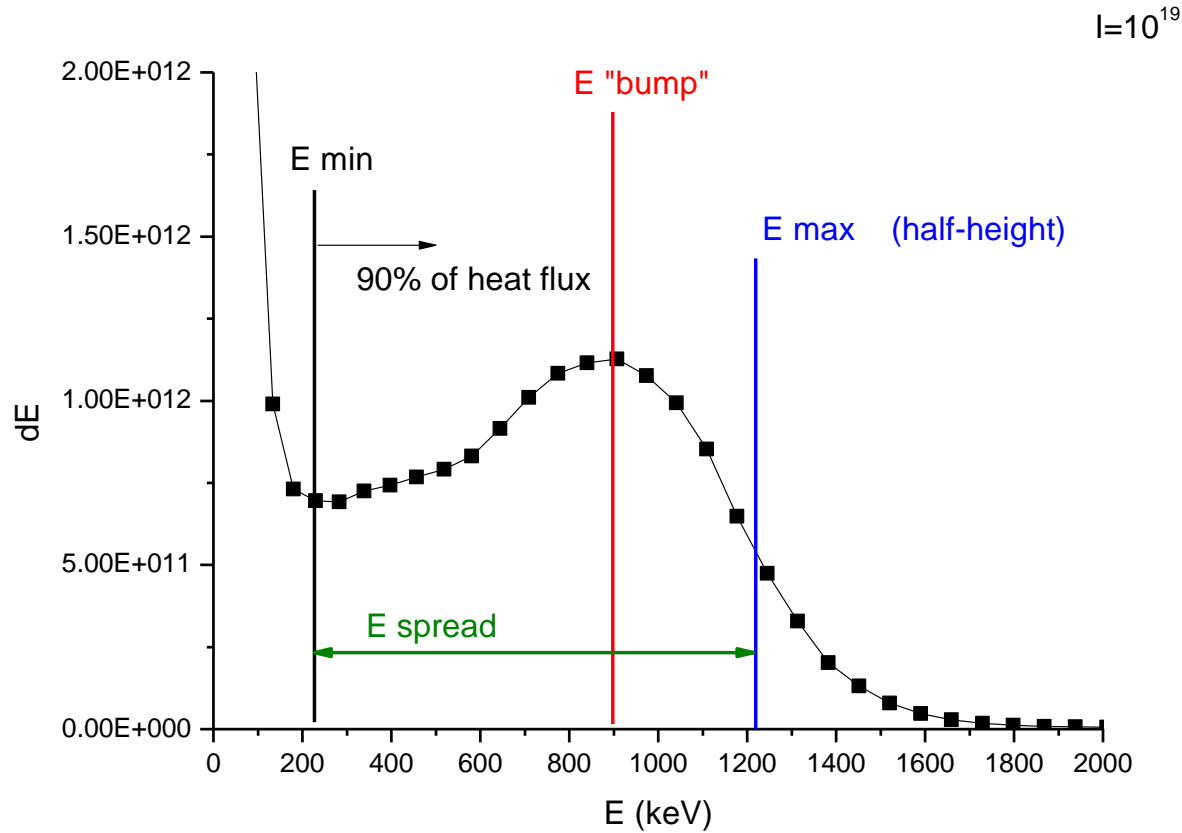
$$n = \int \frac{dn}{dE} dE$$

$$\langle \varepsilon \rangle = \int g(E) dE$$

↑  
the “energy function”

Na levo je angle-averaged dist. func. plotted in the usual way i.e. as  $dn/dE$  whose height gives the number of electrons in that energy bin. Kao mozete videti fitting straight lines to this dist. and attempting to infer a temperature is somewhat arbitrary. If instead we plot the “energy” func. defined in **this** way then we can immediately tell which energy bins contrib. signif. to the total energy density.

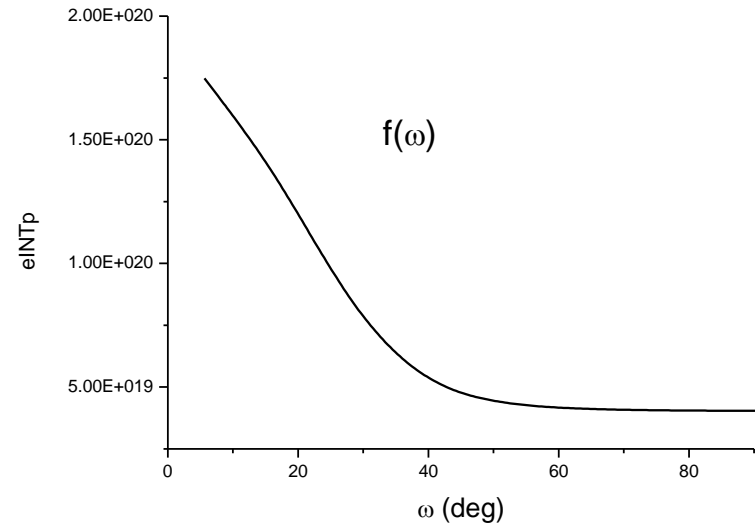
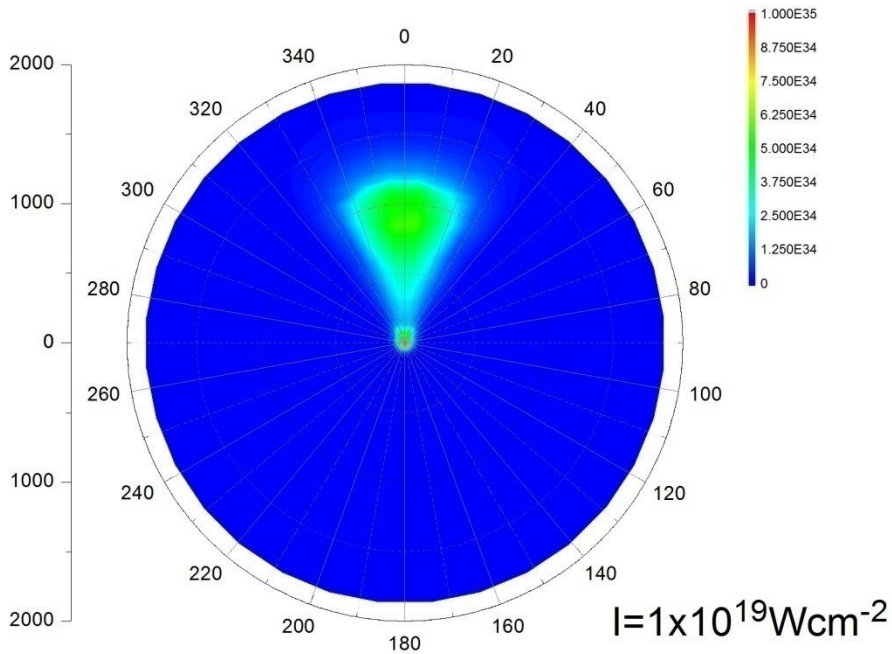
# Characterising the Angle-Averaged $f(E)$



The aim is to characterise the dist. func. in terms of the “energy func.” and look at its characteristics as a function of inten. Here is an actual energy function taken from the simulations at  $10^{19}$ . The characteristics of most interest are the energy peak (or bump), the average energy, the min. E defined as the energy above which 90% of the E flux is captured; the max E def. as the half-height and the energy spread given by the green line.



# Angular space?

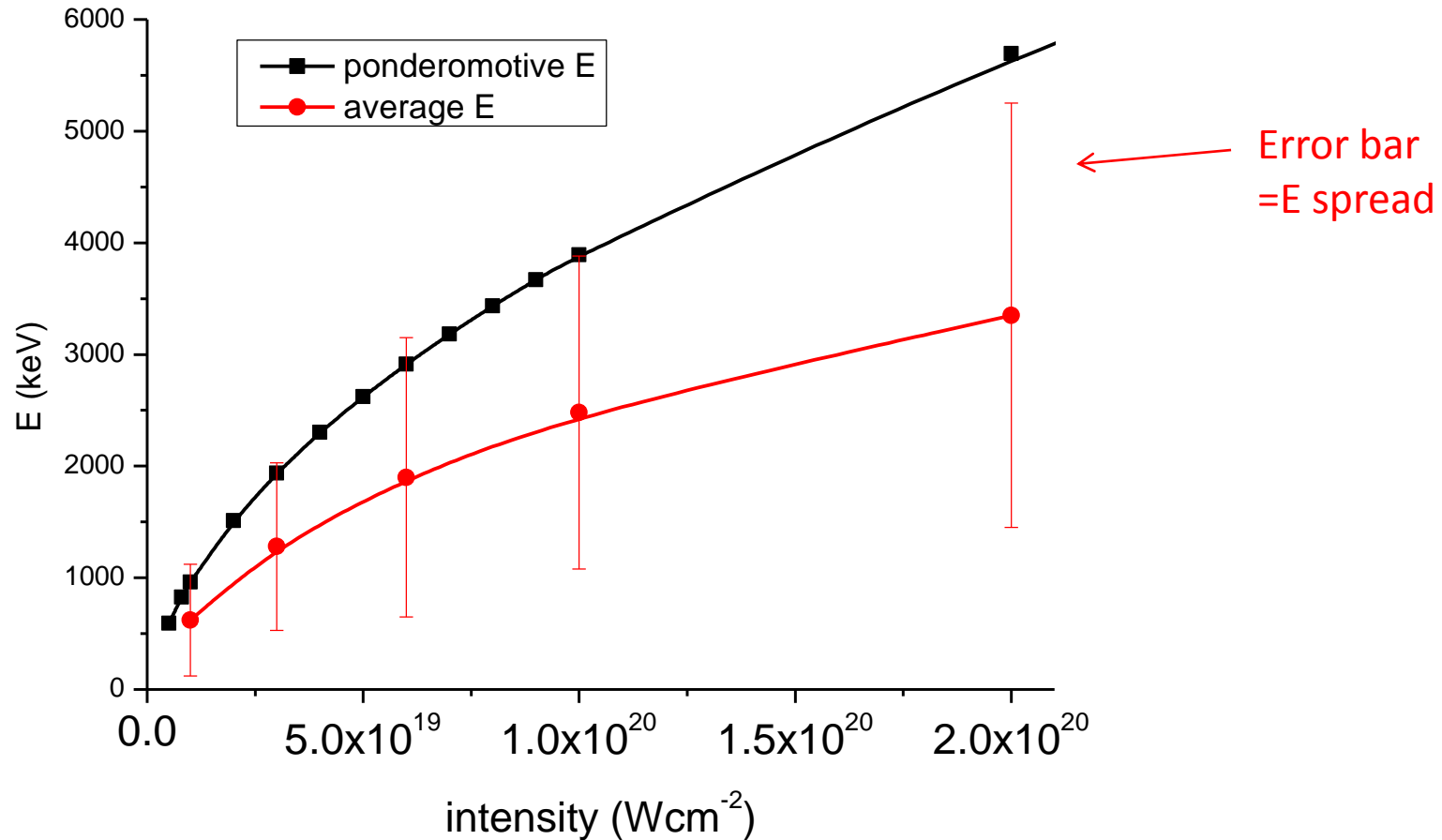


Angular part of  $f(\mathbf{p}) = f(E)f(\omega)$  is well represented by:

$$f(\omega) = e^{-[\omega/\omega_{half}]^4}$$

Nisam jos spomenuo angular space. Na levo je the full energy func. in polar coordinates for  $10^{19} \text{ W cm}^{-2}$ . If we average over energy and plot the resulting function as a function of the angular coordinate then we get the plot on the left, which turns out to be very well represented by a function of this form. N is chosen to match the half-height half-angle in the plot.

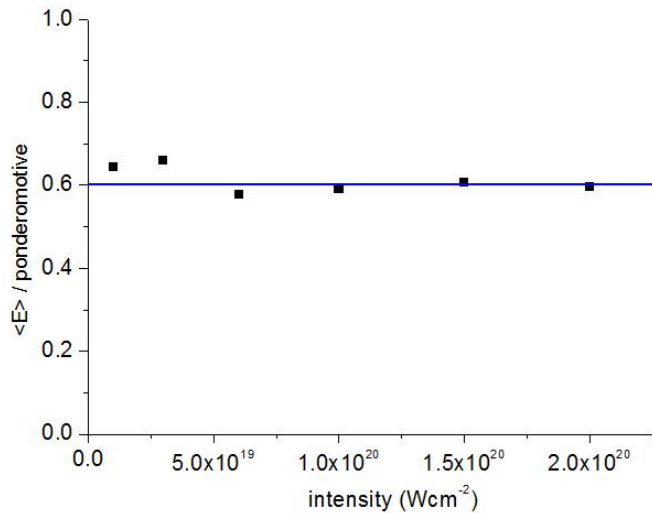
# Hot electron energy scaling



Intensity for  $\langle E \rangle = 1 \text{ MeV}$  is about  $2 \times 10^{19} \text{ Wcm}^{-2}$ .

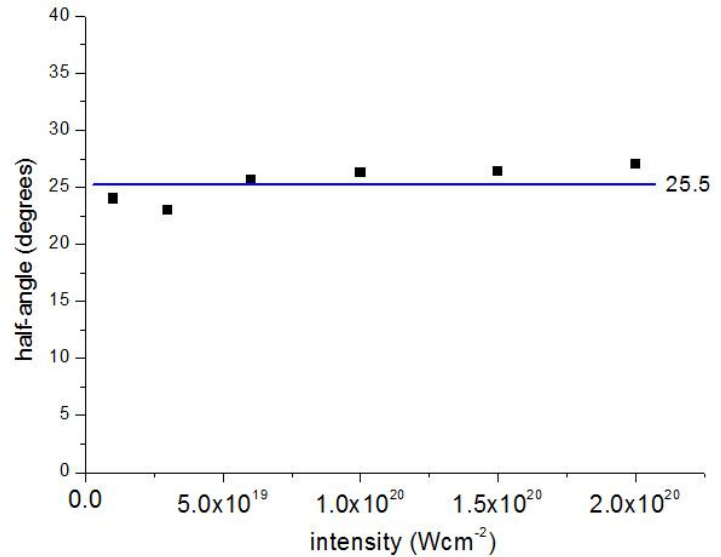
Pa prijede vam formula za distribuciju funkcije, predstavljajući vam neke od njenih osnovnih karakteristika. Najzanimljivije je skaliranje prosečne energije sa intenzitetom, što je crvena linija. Crna linija predstavlja ponderomotivnu energiju. Greške na rezultatima simulacije predstavljaju rasipanje energije funkcije. Kao što možete videti, prosečna energija je nešto niža od ponderomotivne energije. Koliko niže tačno?

# Characterising f(E)



Average energy scales in same way as ponderomotive.

$$\frac{\langle E \rangle}{E_{\text{pond}}} \approx 0.60$$



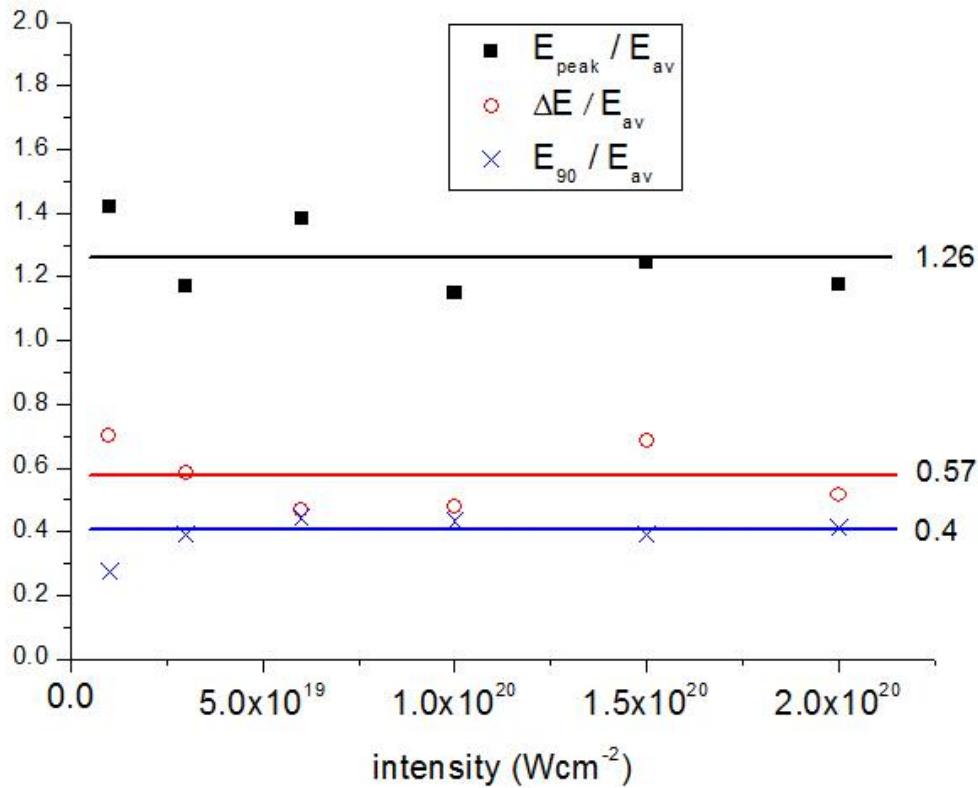
Half-angle independent of intensity: about 25.5deg.

To answer that question you can plot the ratio of the average energy to pond. energy (as a func of inten.) and the ratio is surprisingly constant at around 0.6

...

Sleuci characteristic of interest is the half-height half-angle which is also surprisingly constant at about 25.5deg. These two facts already make it easier to obtain a formula for  $f(p)$ , but it gets even easier...

# Characterising f(E)



Generally true

$$E_{spread} \approx 0.57 E_{peak}$$

$$E_{peak} \approx 1.26 \langle E \rangle$$

$$E_{90} \approx 0.4 \langle E \rangle$$

# Fit to shifted-Gaussian

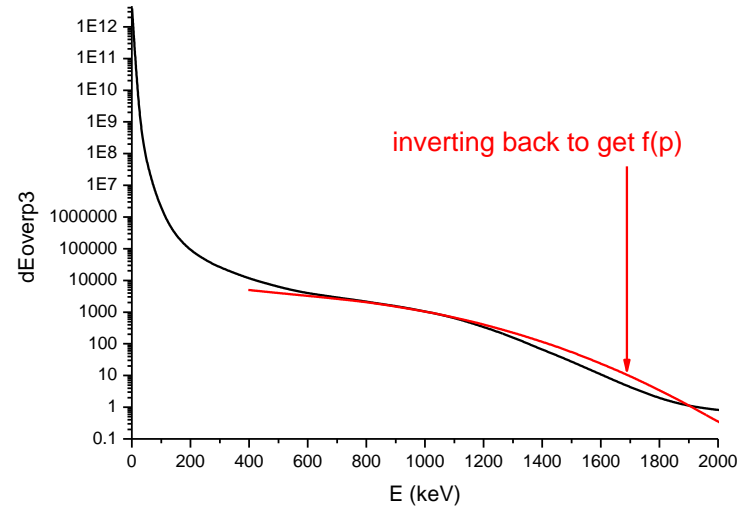
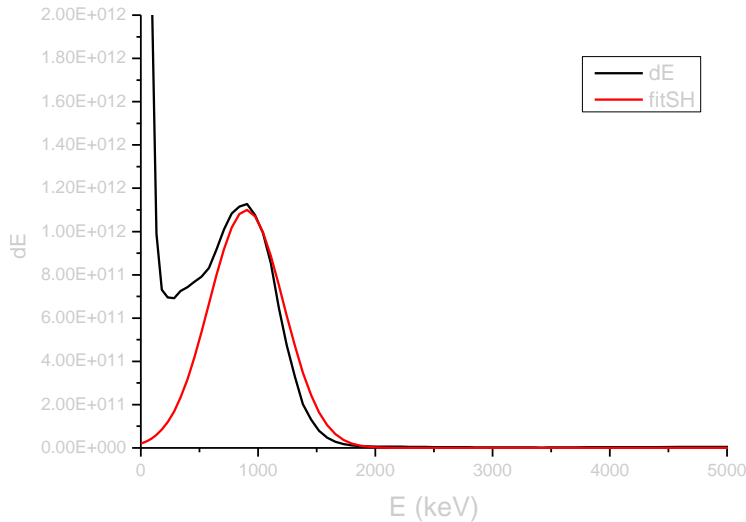
$$f(E) = f(E) f_{ang}(\omega) = c_{norm} \exp\left\{-\left(\frac{E - E_{av}}{\Delta E}\right)^2\right\} \exp\left\{-\left(\omega / \omega_{half}\right)^4\right\}$$

where

$$p_{sh} = 0.6 \frac{E_{osc}}{c}$$

$$\omega_{half} = 25.5^\circ$$

$$f(p < 0.4 p_{av}) = 0$$



A good fit to the energy function is found with a shifted-Gaussian so this is the final form for the fast electron dist. which is just found approximately by dividing the energy function by  $p^3$ . The parameters needed for the fit are easily found from the intensity via these formulae. Since this form for  $f$  diverges at small  $p$  we need to bear in mind that it's only valid for momenta greater than that corresponding to the minimum  $E$  mentioned earlier (and this turns out to be always about 0.4 of the peak  $p$ ). The plot on the right shows what happens when you invert back to get  $f$  – the red line fits  $f$  in just the right place...

# Simple models

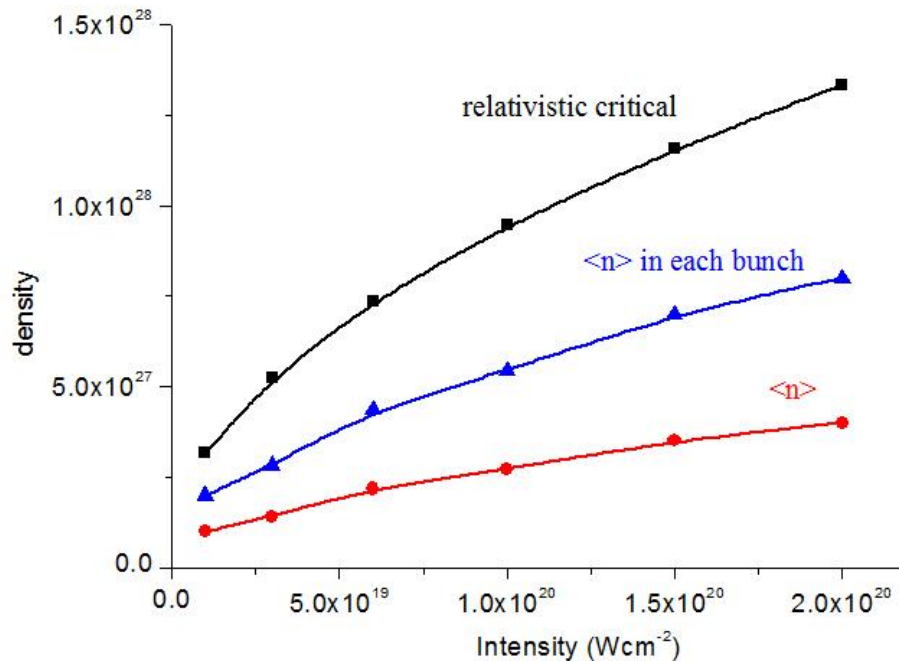
Treating the fast electrons as a fluid, energy balance is:

$$\alpha_E I = n_{fast} \langle E \rangle v_{fast}$$

$$\alpha_p \frac{I}{c} = n_{fast} \langle p \rangle v_{fast}$$

It is tempting to set  $n_{fast}$  = relativistic critical density, but this is not true (although it has the same scaling):

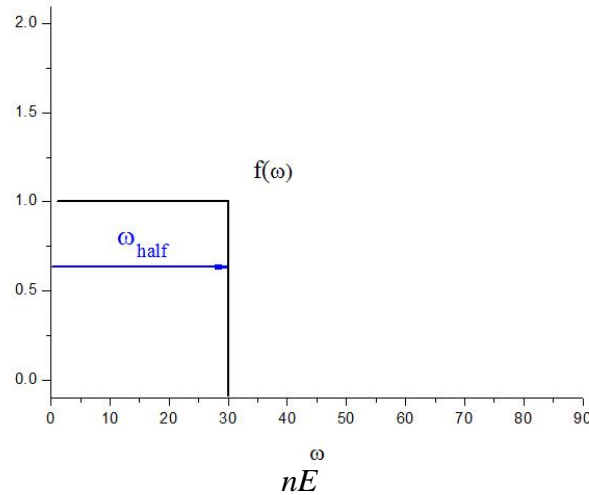
This is important because  $n_{fast}$  determines the current transported:  $j = n_{fast} v_{fast}$



# Why 25.5deg ?

Imagine simple form in angular coordinate:

$$f(\omega) = \frac{U(\omega, \omega_{half})}{2\omega_{half}}$$



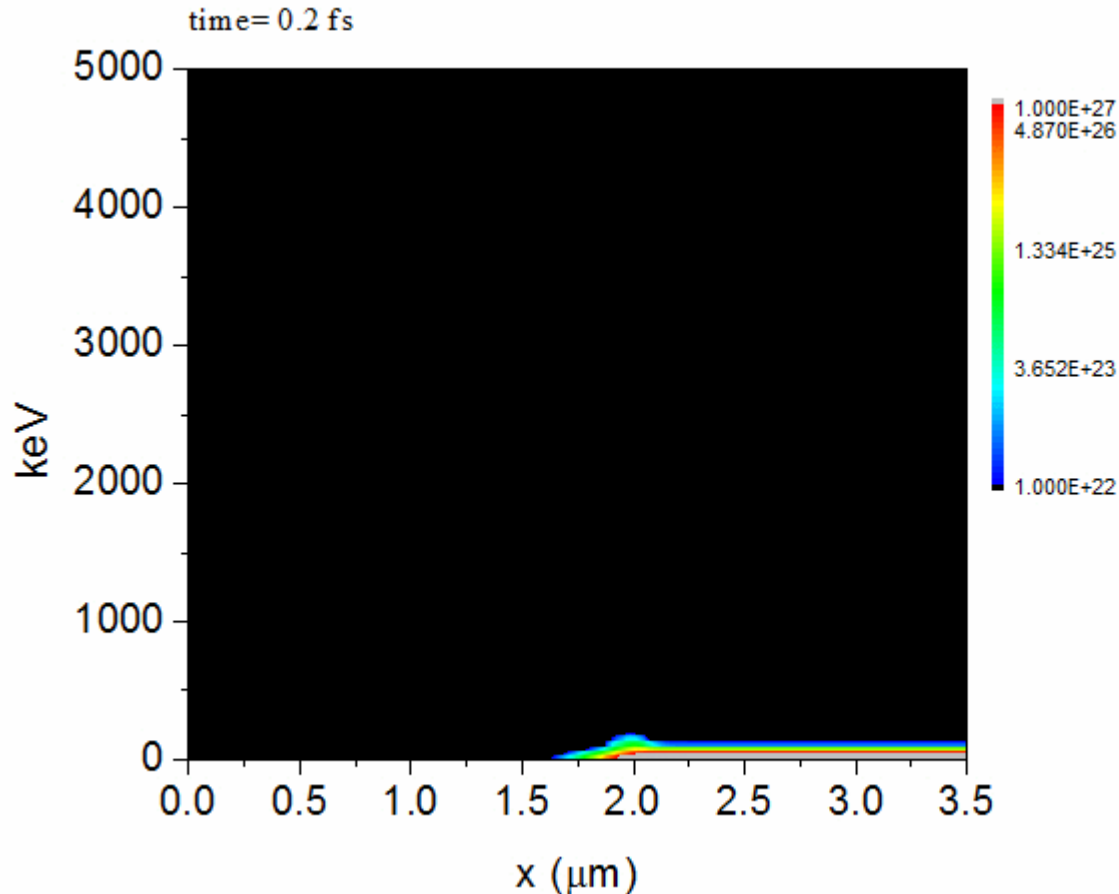
Energy balance:  $0.9\alpha I = \langle nEv_x \rangle \simeq \langle nEv \rangle \langle \cos(\omega) \rangle$

$$\int_0^\infty Ev(E)g_E(E)dE \int_0^{2\pi} \cos\theta \frac{U(\theta, \theta_0)}{2\theta_0} d\theta = 0.9\alpha I$$

Average over all intensities:

$$\langle \omega \rangle = 23.5^\circ$$

# Example: $I=1 \times 10^{19} \text{ W cm}^{-2}$ phase space



- Fast electrons generated at 2w.
- Absorbed electrons are generated locally (they don't sample large parts of the laser wave fields).
- Fast electrons seem to lose some energy as they flow into the target.....why?



# Vacuum Energy = Ponderomotive ?

Yes, but why?

$$E_y = \sqrt{2I / \epsilon_0 c} \quad I = 10^{19} \text{Wcm}^{-2} \quad E_y = 8.7 \times 10^{12} \text{Vm}^{-1} \quad E_y \approx 1.6 \times 10^{13} \text{Vm}^{-1}$$

Travelling wave

Simulation

$I$

$(1 - \alpha)I$

$$E_y \rightarrow (1 + \sqrt{1 - \alpha})E_y = 1.6 \times 10^{13} \text{Vm}^{-1}$$

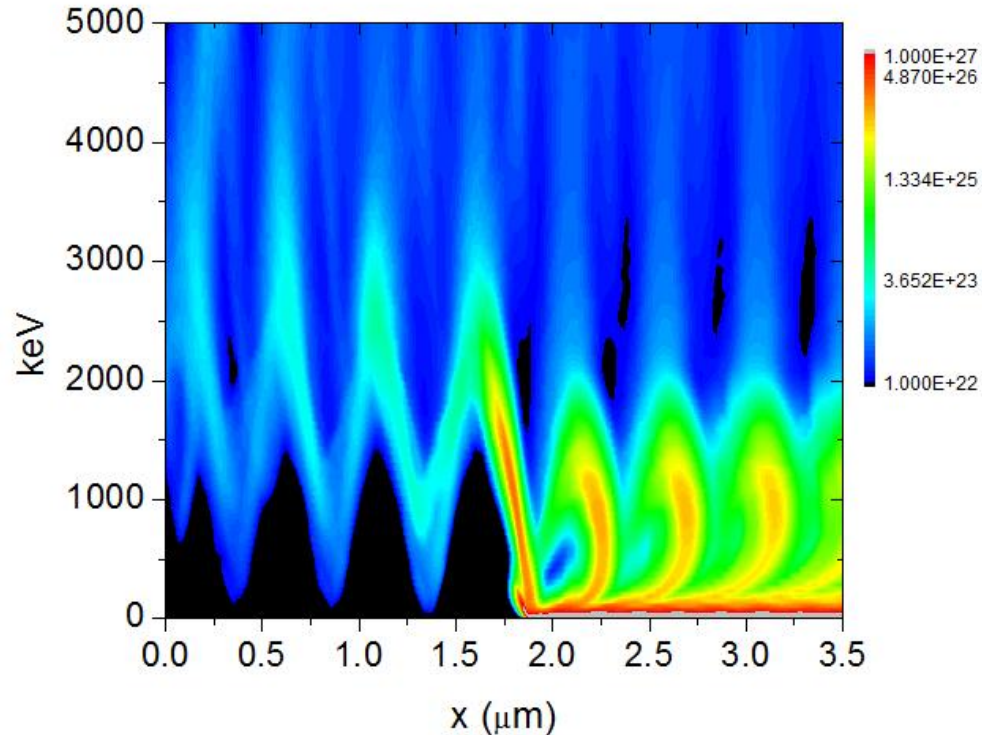
Incident

Reflected

Superposition

$$E_{vac} \approx (1 + \sqrt{1 - \alpha})E_{osc} \approx 2E_{osc}$$

# Vacuum Energy = Ponderomotive ?



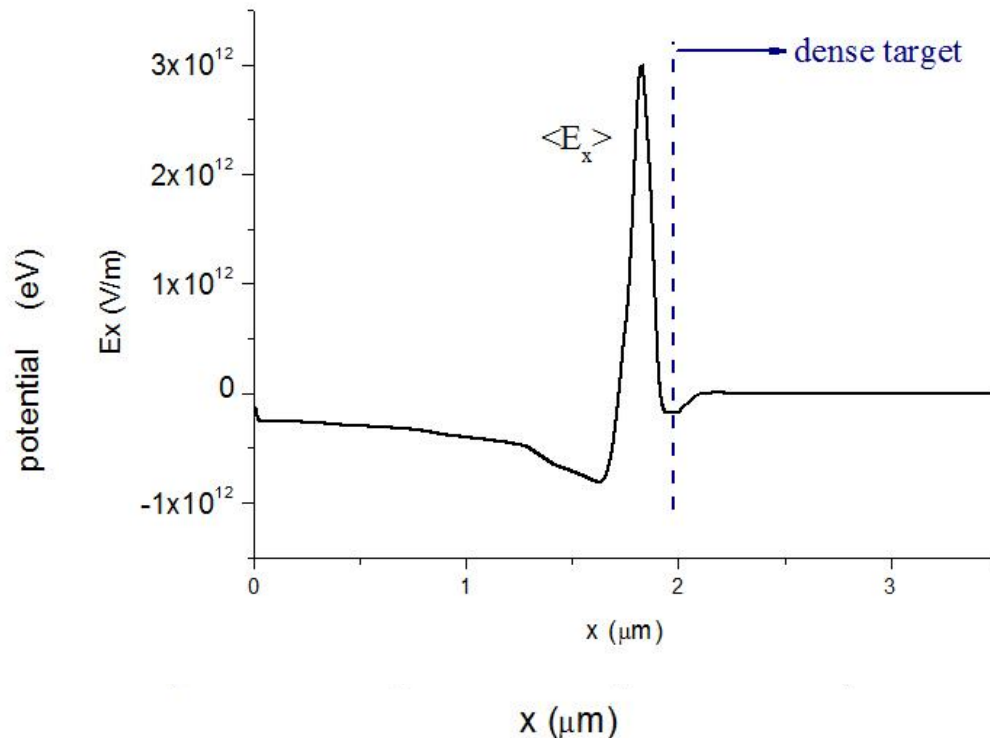
Electrons at front of return-pulse travel out  $\sim 1/4$  wavelength and gain  $\sim$ twice the ponderomotive energy; electrons at the back gain  $\sim$ nothing.

$$\langle E_{vac} \rangle \approx \frac{1}{2} (1 + \sqrt{1 - \alpha}) E_{osc} \approx E_{osc}$$

# Potential Drop

Given vacuum electrons gain ~ponderomotive energies, why are the absorbed electrons at a lower energy?

A primary candidate for the energy extraction is the longitudinal electrostatic field.



The field exists to drive the return current.

$$\Delta V \approx 0.4 E_{osc}$$

# How does the longitudinal field evolve?

Evolution of  $\mathbf{j}$  in terms of  $\mathbf{m}$

$$\frac{\partial \mathbf{j}}{\partial t} = \frac{-e}{\gamma m_e} \left\{ \frac{\partial \mathbf{m}}{\partial t} \right\} - \frac{\mathbf{j}}{\gamma} \frac{\partial \gamma}{\partial t}$$

Fluid equations of motion  
in terms of current:

$$\frac{\partial \mathbf{j}_c}{\partial t} = \frac{-e}{\gamma_c m_e} \left\{ -en_c \mathbf{E} \right\} - \frac{\mathbf{j}_c}{\gamma_c} \frac{\partial \gamma_c}{\partial t}$$
$$\frac{\partial \mathbf{j}_f}{\partial t} = \frac{-e}{\gamma_f m_e} \left\{ -en_f \mathbf{E} + \mathbf{f} \right\} - \frac{\mathbf{j}_f}{\gamma_f} \frac{\partial \gamma_f}{\partial t}$$

Wave-Equation for E

$$\frac{\partial}{\partial t} (\mathbf{j}_c + \mathbf{j}_f) = \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Ohm's Law

$$\varepsilon_0 \mathbf{E} = \left( \omega_{pc}^2 + \omega_{pf}^2 \right)^{-1} \left\{ \frac{e}{m_e} \frac{\mathbf{f}}{\gamma_f} + \left( \frac{\mathbf{j}_c}{\gamma_c} \frac{\partial \gamma_c}{\partial t} + \frac{\mathbf{j}_f}{\gamma_f} \frac{\partial \gamma_f}{\partial t} \right) \right\} + \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

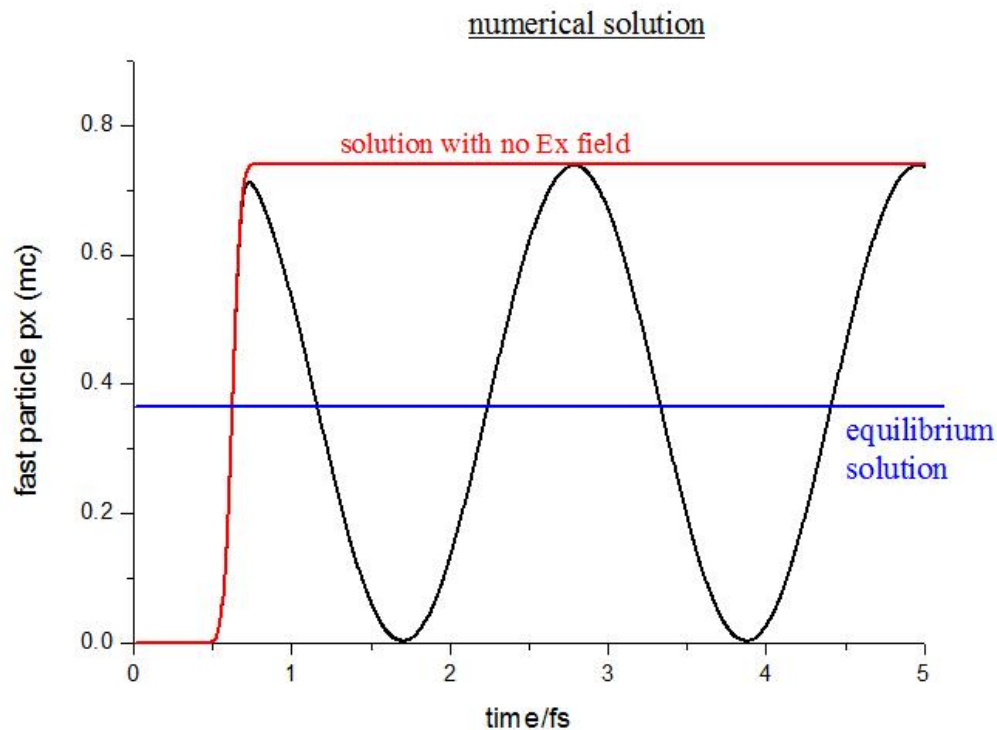
Equilibrium solution

$$\mathbf{E} \approx \frac{n_f}{n_c + n_f} \mathbf{f} = \frac{1}{2} \mathbf{f}$$

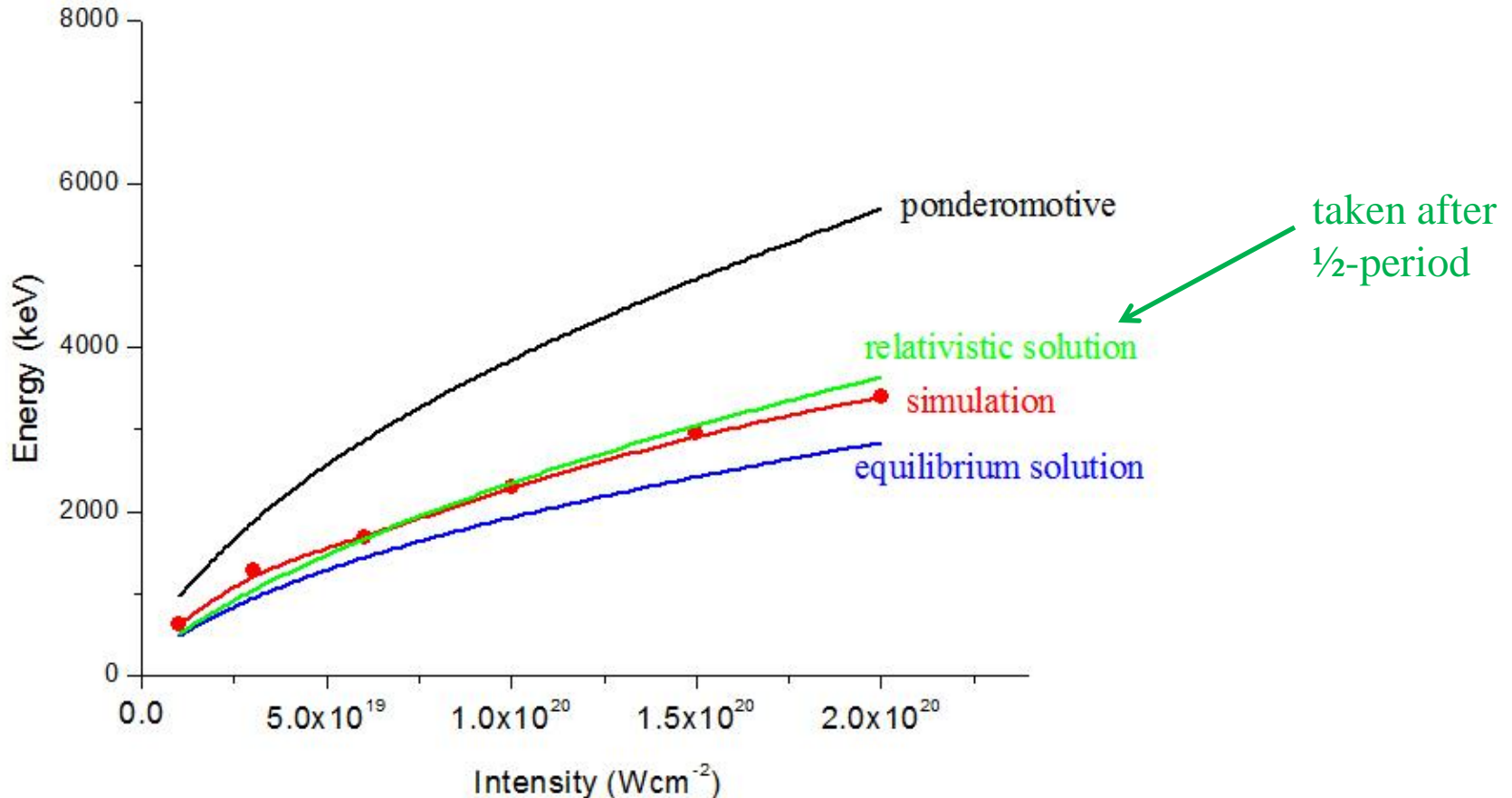
# How does the longitudinal field evolve?

Relativistically, strong plasma oscillations are induced by the external force.

How long does the plasma have to respond to these oscillations? Only  $\frac{1}{4}$  of a wave-period.



# Hot electron energy scaling

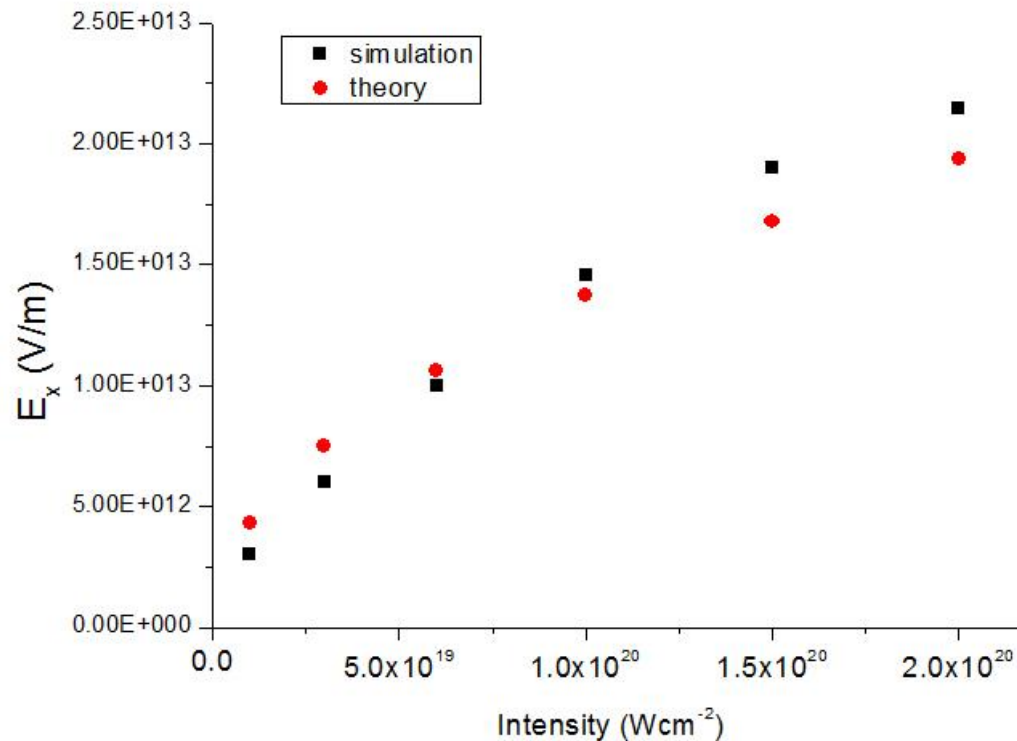


Fast electron current scales as  $I^{1/2}$  so fast electrons always lose energy in proportion to that which they have (also  $I^{1/2}$ ) in order to drive the return current.

# The peak longitudinal field

This field drives ion-acceleration and leads to profile-steepening.

$$\max(\mathbf{E}_L) = \frac{1}{2} \mathbf{j} \times \mathbf{B}$$



# Summary

- Simple formula for  $f(E)$  in terms of  $I$ .

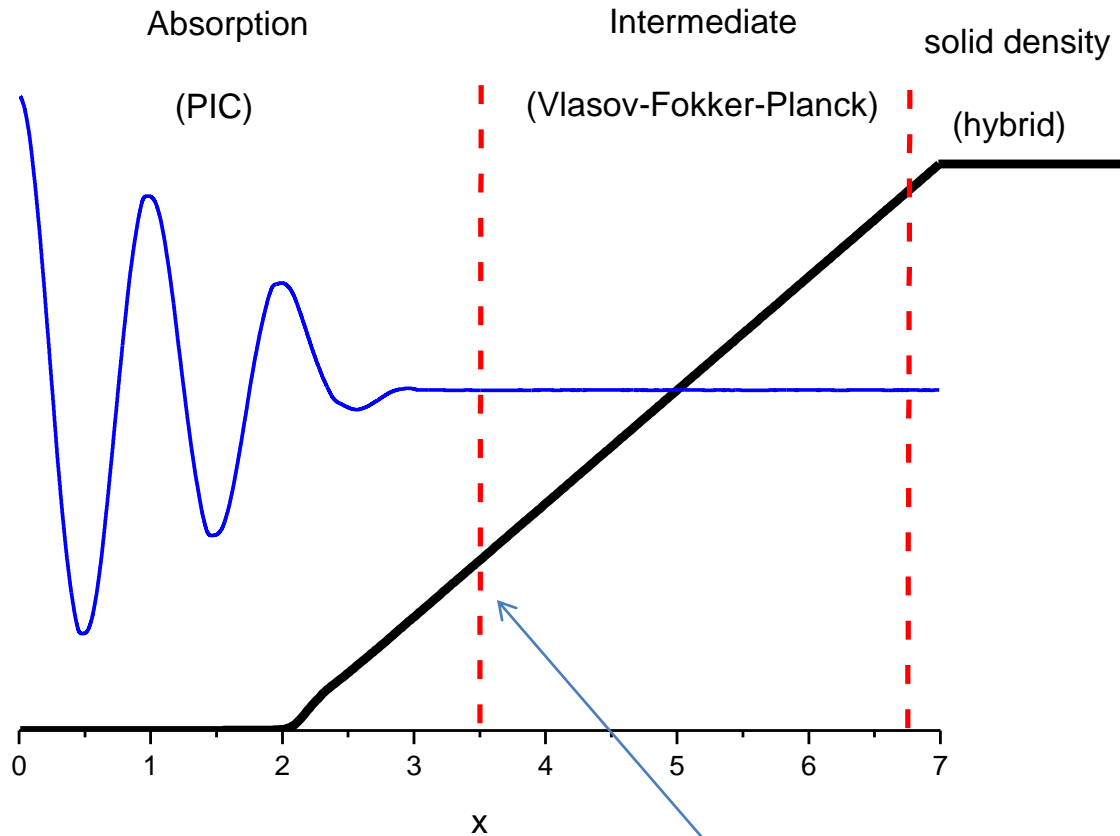
$$f(E, \omega) = c_{norm} \exp\left\{-\left(\frac{E - E_{av}}{\Delta E}\right)^2\right\} \exp\left\{-\left(\omega / \omega_{half}\right)^4\right\}$$

- Energy transferred to the return current reduces the fast-electron energy.

$$\langle E \rangle \approx 0.6E_{osc}$$



# The Intermediate Regime

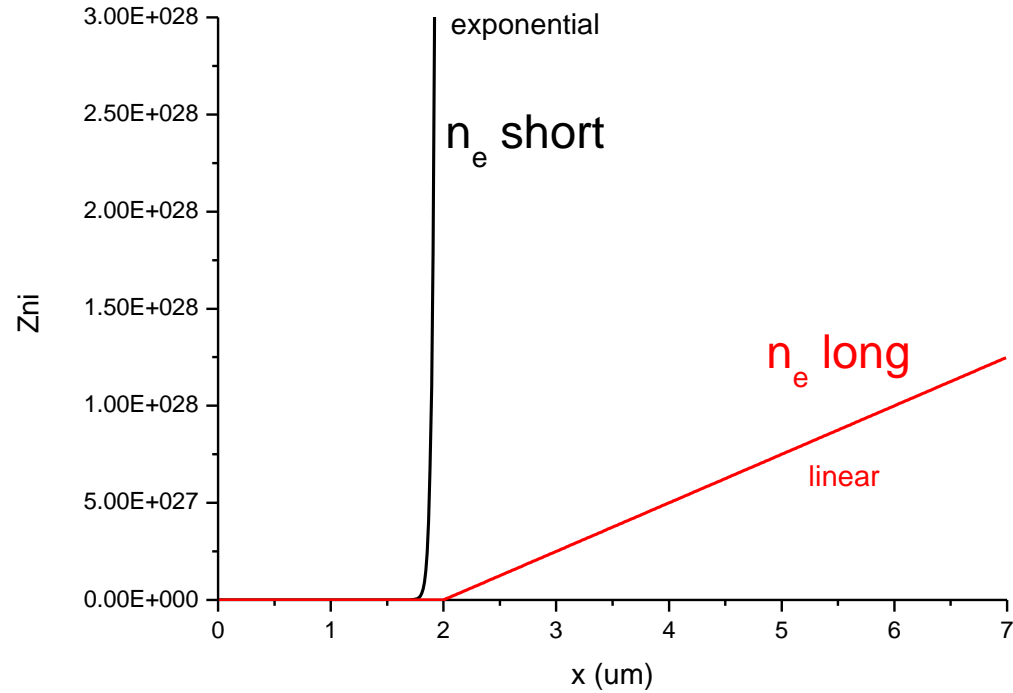


Source of fast electrons?

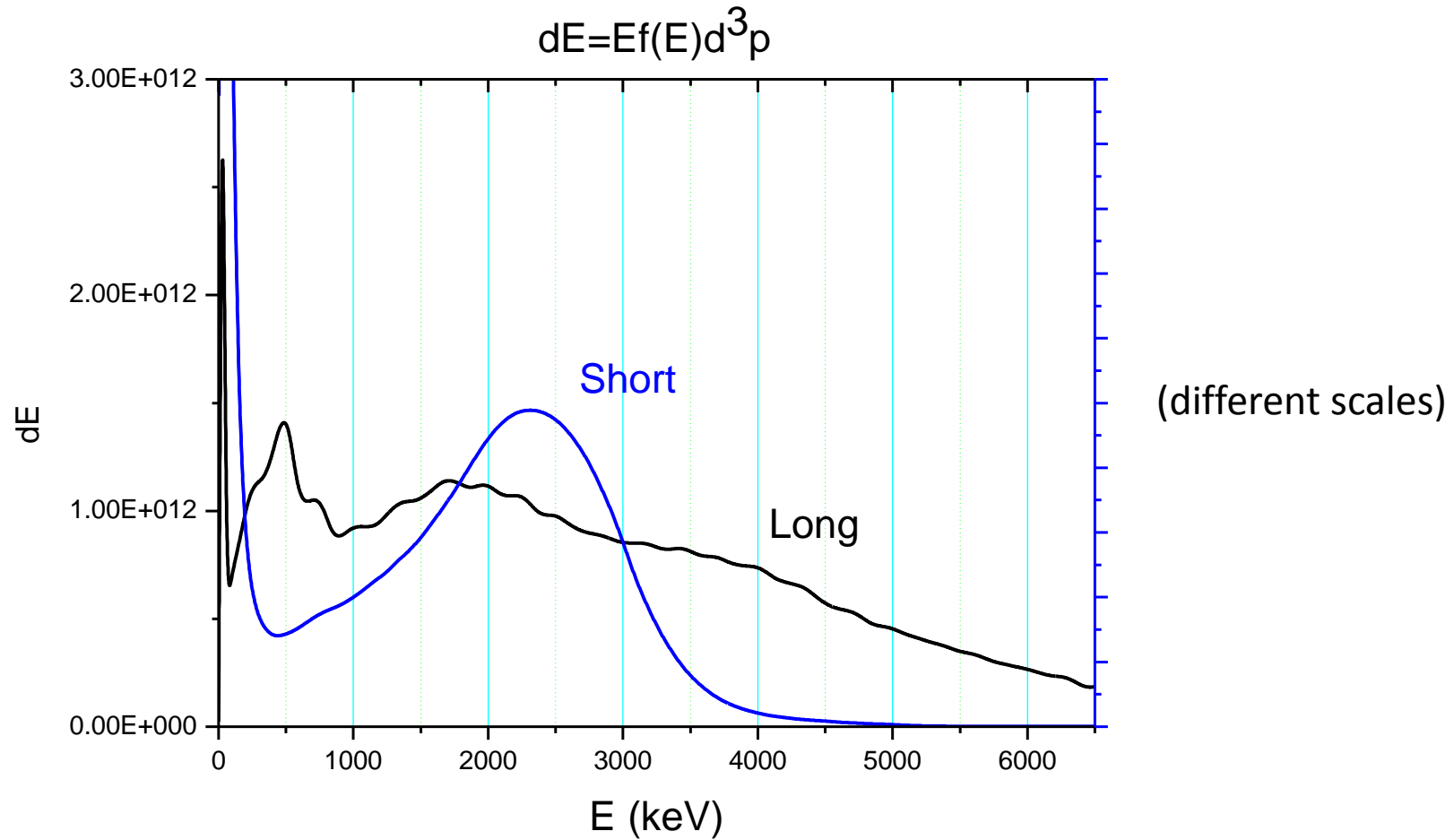
# What is E in a collisionless plasma?

- Usually the background plasma is collisional and we are interested in phenomena which occur on timescales longer than  $t_{\text{coll}}$ . This allows us to know the background momentum eqn on timescales  $\gg t_{\text{coll}}$  and hence obtain E or j.
- In low density hot plasma  $t_{\text{coll}}$  becomes long so we don't know how the background responds. In these circumstances the plasma is undamped and large plasma waves are possible. On timescales longer than  $\omega_{\text{pb}}$  we can assume these oscillations are unimportant and average over them. What is left is an eqn relating the electric field to ALL plasma variables i.e. Including the hots. The background is not collisional and therefore responds to these terms.
- In a sense this is not Ohm's Law, because it does not apply to a collisional system.

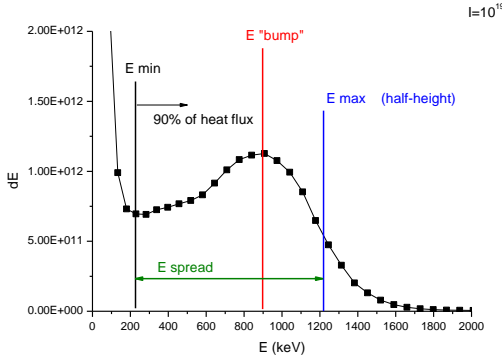
# Long scale-lengths: more complicated



# Long scale-lengths: more complicated



# Fitting $f(E)$



- Don't try to fit  $f(p)$ . Instead fit to  $g(E)$  and invert back to  $f(p)$ .

$$\langle \varepsilon \rangle = \int g(E) dE$$

$$\langle \varepsilon \rangle = \int E(p) f(p) d^3 p = \int E(p) f(p) p^2 \frac{dp}{dE} dE$$

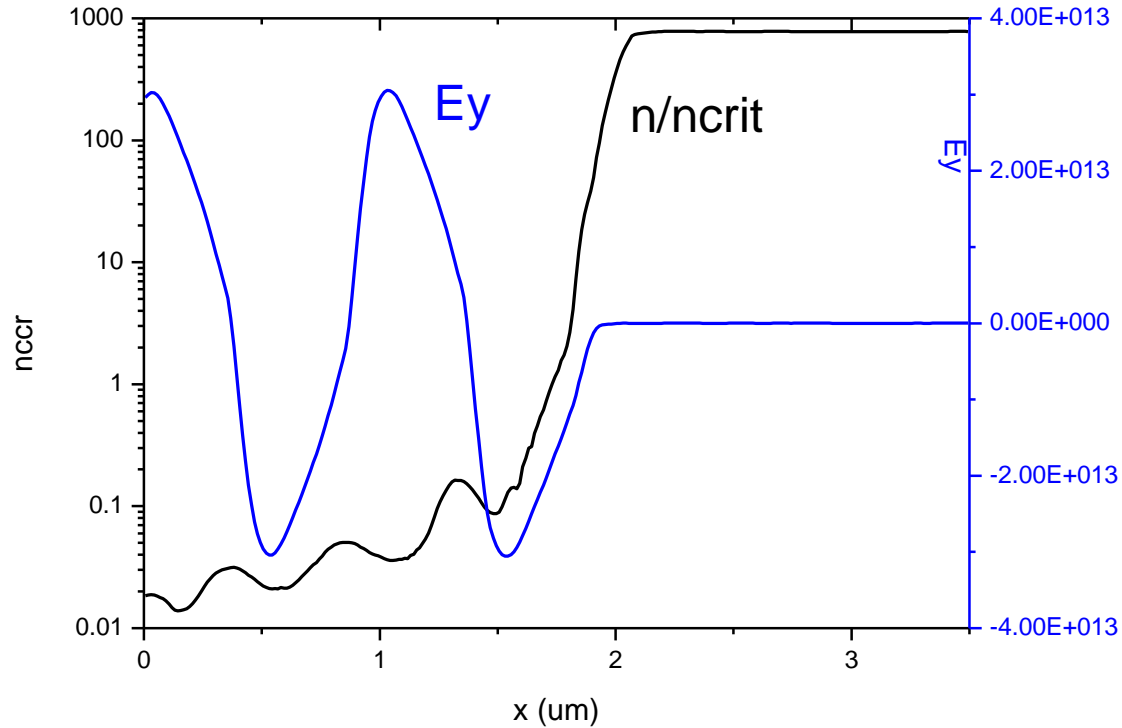
$$= \int E p(E) \gamma(E) f(p(E)) dE$$

$$\Rightarrow f(p(E)) \propto \frac{g(E)}{E \gamma(E) p(E)}$$

Numerically  
integrate

# Example $6 \times 10^{19} \text{Wcm}^{-2}$

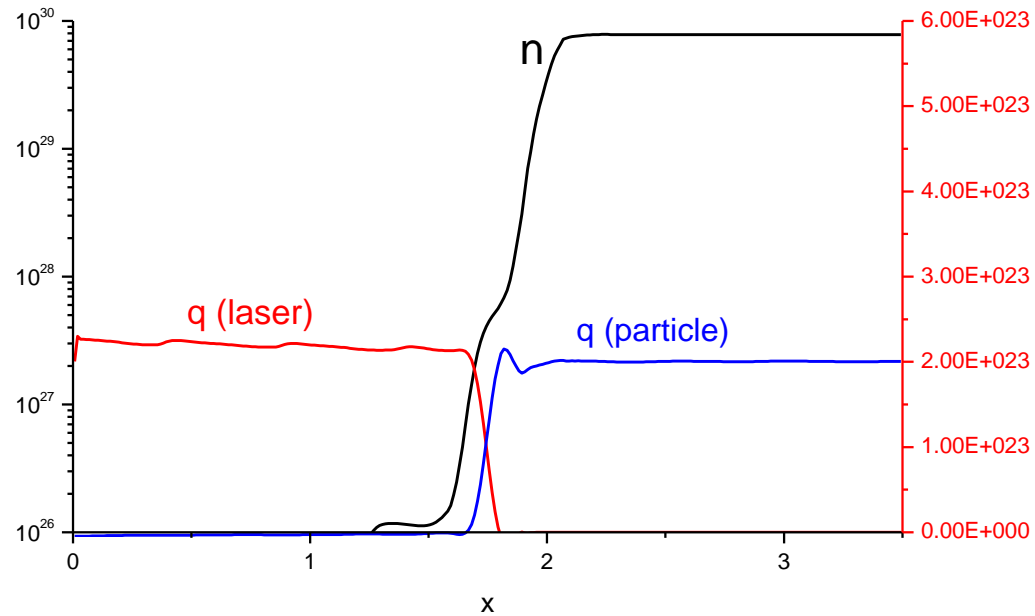
Scale-length= $0.05\lambda$   
normal incidence



Laser (transverse) electric field

# Example $6 \times 10^{19} \text{Wcm}^{-2}$

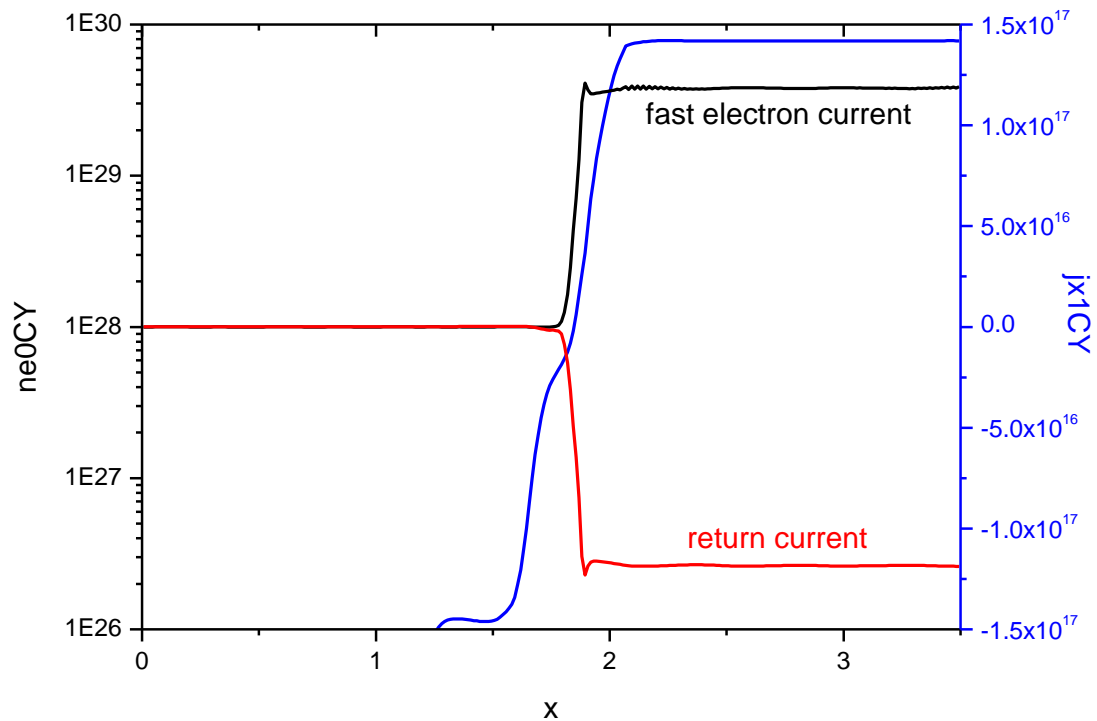
Scale-length= $0.05\lambda$   
normal incidence



Heat flux

# Example $6 \times 10^{19} \text{ W cm}^{-2}$

Scale-length =  $0.05\lambda$   
normal incidence

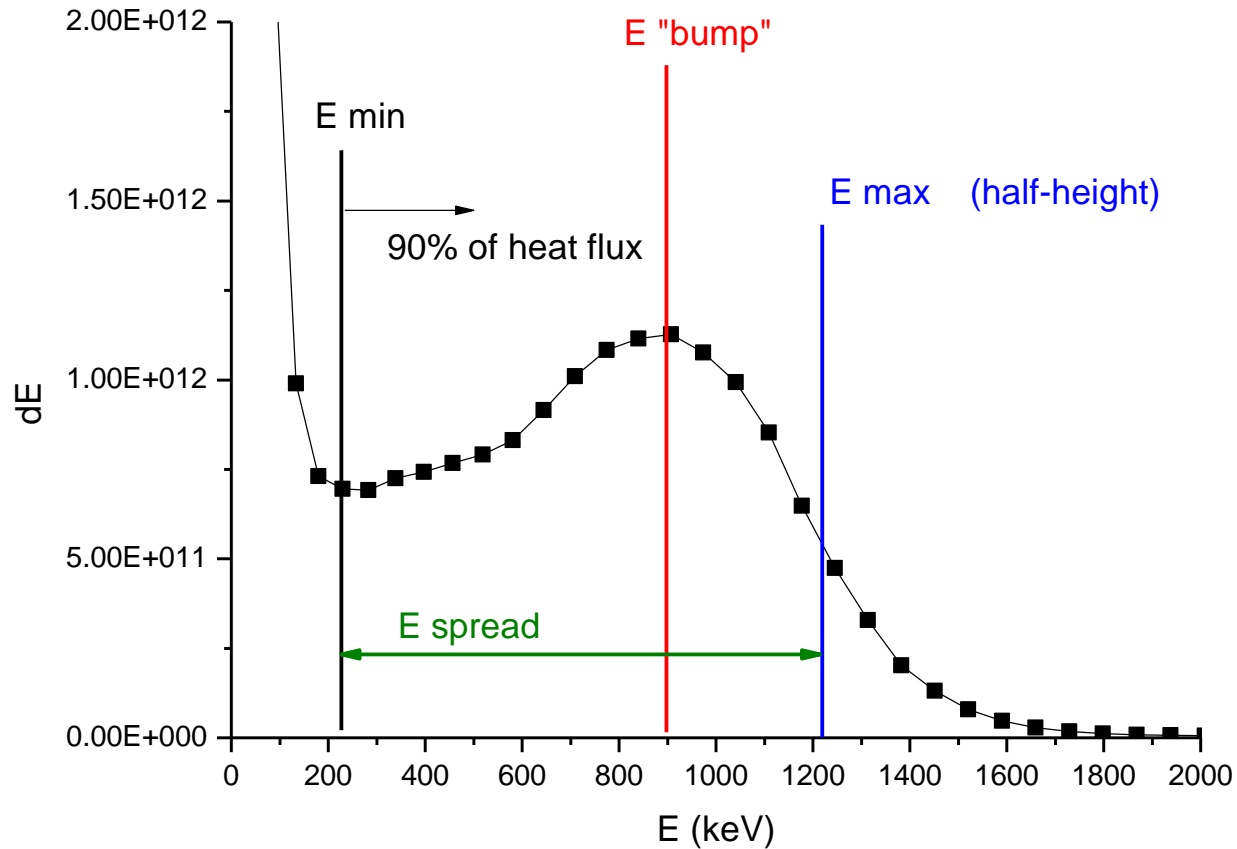


Current density



# Hot electron energy scaling

$I=10^{19}$



Intensity for  $\langle E \rangle = 1 \text{ MeV}$  is about  $2 \times 10^{19} \text{ W cm}^{-2}$ .