The Macrophysics & Microphysics of Cosmic Rays

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Cosmic Rays are Pervasive
Tools & Systems

• Direct probes
  • In situ particle detectors
  • Telescopes from radio to $\gamma$-ray
  • $\nu$ detectors

• Indirect probes
  • Astrochemical modeling
  • Geology

• Natural and lab plasmas
  • Space & solar
  • Diffuse and HED
Properties

Energy Spectrum

- A broken power law:
  \[ N(E) \sim E^{-2.7}, \quad E_{\text{PeV}} < 3 \]
  \[ \sim E^{-3.0}, \quad 3 < E_{\text{PeV}} < 100 \]
- Strong solar cycle modulation below \( \sim 10 \) GeV
- Energy density \( \sim 1 \text{ eV cm}^{-3} \), near equipartition with magnetic & thermal/turbulent energy density of interstellar gas.
- Most of the pressure comes from \( \sim \) GeV particles

Highly isotropic & resident for \( \sim 10 \) Myr
(An)Isotropy

Left: The distribution of cosmic ray arrival directions is highly isotropic, up to the knee.
Right: Weak fluctuations at TeV energies have been discovered recently & challenge theory.

Hillas 1984

Abbasi et al. 2010
Cosmic Ray Acceleration

- Curved shock front (red), fluid flow in frame of shock (black arrows), magnetic fieldlines in purple.

NASA image
Diffusive Shock Acceleration

Particles are scattered back and forth across the shock by waves and turbulence they generate themselves, resulting in a power law spectrum that depends on the shock compression ratio.

$E^{-2}$ for strong shock

Maximum $E$ is set by shock evolution & geometry.
Overarching Questions

• How does $10^{-9}$ of interstellar particles come to have as much energy as the thermal pool?
  – Observational probes
  – Acceleration & propagation processes

• How does this virtually collisionless component interact with the thermal gas & magnetic field?
  – Kinetic instabilities
  – Fluid treatments
Fits to the $\gamma$-ray spectrum of M82: Left is best fit to $\gamma$-ray spectrum, Right is best fit to radio spectrum. Milky Way-like assumptions about cosmic ray acceleration produce good fits (Yoast-Hull et al 2013)
Breakdown of Equipartition

**Cosmic Rays in Arp 220**

<table>
<thead>
<tr>
<th></th>
<th>Supernova Power (erg yr(^{-1}))</th>
<th>Average Gas Density (cm(^{-3}))</th>
<th>Cosmic Ray Energy Density (eV cm(^{-3}))</th>
<th>Radiation Field Energy Density (eV cm(^{-3}))</th>
<th>Magnetic Field Energy Density (eV cm(^{-3}))</th>
<th>Magnetic Field Strength ((\mu)G)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Milky Way</td>
<td>(2 \times 10^{48})</td>
<td>1</td>
<td>1.4</td>
<td>0.3</td>
<td>0.9</td>
<td>6</td>
</tr>
<tr>
<td>M82</td>
<td>(7 \times 10^{48})</td>
<td>550</td>
<td>250</td>
<td>1000</td>
<td>1800</td>
<td>250</td>
</tr>
<tr>
<td>Arp 220 East</td>
<td>(7 \times 10^{49})</td>
<td>17,000</td>
<td>1300</td>
<td>40,000</td>
<td>220,000</td>
<td>3000</td>
</tr>
<tr>
<td>Arp 220 West CND</td>
<td>(1.3 \times 10^{50})</td>
<td>90,000</td>
<td>3500</td>
<td>440,000</td>
<td>890,000</td>
<td>6000</td>
</tr>
</tbody>
</table>

**Note.** — Values for the Milky Way are taken from Table 1.5 in Draine (2011). For M82 & Arp 220, the values for the cosmic ray & magnetic field energy densities taken from our best-fit models (see Section 3 above and YEGZ). 

**Yoast-Hull et al. 2015**
Milky Way Central Molecular Zone

(a) Model A, Soft Electron Spectrum
(b) Model A, Soft Proton Spectrum

Soft electron spectrum over produces radio; soft proton spectrum requires huge energy input. From Yoast-Hull et al. 2014.
Coupling to the Thermal Gas

Collisional
- Ionization
  - Heating
  - Chemistry
- Radiative Processes
  - $\gamma$-rays from $\pi^0$ decay & inverse Compton
  - Bremsstrahlung
- Light element production

Collisionless
- Momentum & energy exchange mediated by the ambient magnetic field.

The subject of this talk
Plan of This Talk

• Motivation
• Classical Cosmic Ray Hydrodynamics
• Generalized Cosmic Ray Hydrodynamics
  – Justification for approximate treatments?
• Beyond Alfven Waves
  – Going to extremes
• Unfinished business
Galactic Winds: One Motivation

- Steady, spherically symmetric, pressure driven outflow *a la* Chevalier & Clegg 1985 but extended.

  *(Bustard, EZ, D’Onghia 2015)*

- Relativistic fluid cools more slowly & drives a faster wind.

Cory Cotter & Chad Bustard
Milky Way Wind

Top left: Soft x-ray sky,
Bottom left: Magnetic flux tube geometry.
Top right: Domains of flow, with mass loss rates
Bottom right: Gas temperature with & without cosmic ray heating.

Recent Simulations of a Star-Forming Disk

Perpendicular Dynamics are Easy

Cosmic ray force balance:

$$\nabla \perp P_c = \frac{J_c \times B}{c}$$

Lorentz force on thermal gas:

$$J_g \times \frac{B}{c}$$

$$= \frac{J \times B}{c} - \frac{J_c \times B}{c}$$

Pressure gradient introduced through Lorentz force
Parallel Dynamics are Subtle
Gyroresonant Scattering

Orbits follow fieldlines and short wavelength fluctuations average out.

Gyroresonant fluctuations (Doppler shifted frequency $k v_{\text{parallel}} = \omega_{\text{cr}}$) scatter in pitch angle $\cos^{-1} \mu$.

[Graph showing resonant and nonresonant fluctuations]
Gyroresonant Streaming Instability

\[ \Gamma_{cr} = \frac{\pi^2 q^2 v_A^2}{2c^2} \sum_{\pm} \int \delta(\omega - kv\mu \pm \omega_c) \nu(1 - \mu^2) \left[ \frac{\partial f}{\partial p} + \left( \frac{kv}{\omega} - \mu \right) \frac{1}{p} \frac{\partial f}{\partial \mu} \right] p^2 dp d\mu, \]

- resonance
- damping
- excitation by anisotropy

Simple approximation to the growth rate:

\[ \Gamma_{cr} \sim C \omega_{ci} \frac{n_{cr}}{n_i} \left( \frac{v_D}{v_A} - 1 \right) \]

Here & elsewhere I’m interested in the bulk cosmic rays w \( \gamma \sim 1 \)
Fokker – Planck (F-P) Equation

Back reaction of waves on zero order cosmic ray distribution function \( f_0 \)

\[
\frac{df_0}{dt} = - \left\langle \frac{q}{m} \left( E_1 + \frac{v \times B_1}{c} \right) \cdot \nabla_p f_1 \right\rangle = \nabla_p \cdot D \cdot \nabla_p f_0.
\]

Pitch angle scattering \( (D_{\mu\mu}) \) dominates:

Scattering frequency \( \nu \sim \omega_c (\delta B/B)^2 \)

\( D_{p\mu} = D_{\mu p} \) are order \( (v_A/c) \) \hspace{1cm} \( D_{pp} \) is order \( (v_A/c)^2 \)
Energy Equation

Multiply F-P eqn. by particle energy $\varepsilon$ & integrate over momentum space:

$$\frac{\partial U_c}{\partial t} + \nabla \cdot \mathbf{\tilde{W}}_c = - \int d\omega dk 2\Gamma_{cr}(\omega, k) I(\omega, k)$$

- Energy density
- Energy flux
- Energy transfer to waves
Frequent Scattering Approximation

Relate anisotropy to spatial gradient:

\[ D_{\mu\mu} \frac{\partial f_0}{\partial \mu} + D_{\mu p} \frac{\partial f_0}{\partial p} = -\frac{\nu(1 - \mu^2)}{2} \frac{\partial f_0}{\partial z} \]

Energy equation simplifies to:

\[ \frac{\partial U_c}{\partial t} + \nabla \cdot \tilde{W}_c = \nu_A \cdot \nabla P_c. \]

“frictional heating”
Equation for Waves

From Dewar’s theory:

\[
\frac{\partial}{\partial t} \frac{\delta B^2}{4\pi} = -\nabla \cdot W_w + u \cdot \nabla \frac{\delta B^2}{8\pi} - v_A \cdot \nabla P_{cr} - G.
\]

\[
W_w \equiv \frac{\delta B^2}{4\pi} \left( v_A + \frac{3}{2} u \right)
\]

- Adiabatic terms
- Driving & dissipation
- Heating rate

\[
\rho \frac{dQ}{dt} = G.
\]
Fluid Treatment

• “Classical Cosmic Ray Hydrodynamics (CCRH)
  – Equations developed by Volk & collaborators based on self-confinement model
  – Stream down pressure gradient at $v_A$ relative to thermal gas (care needed in implementing this).
  – Transfer momentum through pressure gradient
    - Heat gas at $-v_A \nabla P_c$.
    - Diffusion along $B$ with diffusivity $\kappa \sim \nu^2/\nu$

• Applied to shocks, galactic winds, ISM heating, intracluster medium.
Approximations & Improvisations

• Include $\text{grad } P_c$, advect with fluid, ignore heating, diffusion if any is isotropic.
  – No magnetic field calculation necessary *(implicitly stochastic on gyroradius scale).*
  – No need to ensure streaming is down $\text{grad } P_c$.

• Include streaming relative to thermal gas & frictional heating, but replace $v_A$ by thermal sound speed $v_S$.
  – Same advantages as previous bullet.

These are the main variants in the literature
Generalized Cosmic Ray Hydrodynamics (GCRH)

• Account for non-cosmic ray sources of waves.

• Generalize F-P equation to include waves traveling in both directions.

MHD turbulence, Boldyrev group
From Fokker-Planck Equation

- Composite streaming velocity

\[ w \equiv \frac{v_+ - v_-}{v_+ + v_-} v_A \]

- Pressure gradient force is unchanged
Wave Evolution Equations

\[
\frac{\partial (\delta B^2, \pm)}{\partial t} \frac{1}{4\pi} = -\nabla \cdot W^\pm_w + u \cdot \nabla \frac{\delta B^2, \pm}{8\pi} \pm \frac{v^\pm}{v_+ + v_-} v_A \cdot \nabla P_{cr} - G^\pm + L^\pm.
\]

\[
W^\pm_w \equiv \frac{\delta B^2, \pm}{4\pi} \left( \pm v_A + \frac{3}{2} u \right).
\]
Balance Driving & Damping

Simple model

\[ \tau_A \equiv -\frac{P_{cr}}{\nu_A \cdot \nabla P_{cr}} , \]
\[ \frac{E_+}{E_+ + E_-} \frac{P_{cr}}{\tau_A} - 2\Gamma E_+ + \frac{\dot{E}}{2} = 0 , \]
\[ -\frac{E_-}{E_+ + E_-} \frac{P_{cr}}{\tau_A} - 2\Gamma E_- + \frac{\dot{E}}{2} = 0 . \]

This is easily solved
Transport Velocity

\[ w = \frac{v_A}{x + \sqrt{1 + x^2}} \]

\[ x \equiv \frac{\dot{E} \tau_A}{2P_c} \]

\( w \to 0 \) when external driving dominates
\( w \to v_A \) when cosmic ray driving dominates

Cosmic ray heating is reduced but compensated by turbulent damping
Extrinsic Turbulence Model

- Advect cosmic rays at the fluid speed $v$.
- Neglect cosmic ray heating.
- Retain pressure gradient force.

All hold in the limit of strongly driven, balanced turbulence.

- But, Alfven turbulence produces anisotropic, field aligned diffusion.
- Isotropic diffusion is produced by a small scale, stochastic field.
Beyond Alfven Waves – High $\beta$

- For $\beta = P_G/P_M \gg 1$
- Affects waves which scatter cosmic rays with $\mu > \mu_c$
- Demands very weak fields, e.g $B < 10^{-12}G$ in galaxy clusters.

\[
\mu_c \sim \frac{v_i}{c} \beta^{1/2}.
\]

Enforces sub-Alfvenic streaming
Nonresonant Instabilities

• When $U_{cr}/U_B > c/v_D$ there is a new, nonresonant instability driven by the electron current that compensates the cosmic ray current.

• Conditions are met at shocks, and possibly in young galaxies.
Diffusive Shock Acceleration

Particles are scattered back and forth across the shock by waves and turbulence they generate themselves, resulting in a power law spectrum that depends on the shock compression ratio.

$E^{-2}$ for strong shock

Maximum $E$ is set by shock evolution & geometry.
Rapid Growth to Nonlinear Amplitude

PIC simulation showing magnetic field growth in a shock layer.

Riquelme & Spitkovsky 2010
Beyond Alfven Speed

• Resonant instabilities enforce sub-Alfvenic streaming & require extremely weak fields.

• Nonresonant instabilities require large cosmic ray fluxes
  – Growth rates comparable to frequency require nonstandard treatments
  – Could be very interesting in weak field situations.
Summary & Prospects

• Cosmic rays appear in diffuse plasmas everywhere, in defiance of thermodynamics.
• They exchange momentum and energy with the background medium, mediated by magnetic fields.
• Advances in observation, computation, & experiment make this a wonderful time to study their acceleration, transport, and feedback.