Cross-field electron diffusion due to the coupling of drift-driven microinstabilities

Ken Hara\textsuperscript{1} and Sedina Tsikata\textsuperscript{2}

1. Plasma Dynamics Modeling Laboratory, Assistant Professor, Aeronautics and Astronautics, Stanford University
2. ICARE, CNRS, Orléans, France
Low-temperature magnetized plasmas are observed in various natural phenomena and engineering devices. The dynamics is highly nonlinear: combination of many different physical (and chemical) processes, including:

- Electron gyration (~GHz)
- Instabilities and turbulence (~MHz, µm)
- Self-organization (~kHz, cm)

Need of high-fidelity computational simulations & validation with advanced experiments (e.g. laser diagnostics)
Low temperature magnetized plasmas

Using applied magnetic fields leads to higher electron temperature and density

- **Multiscale** phenomena
  - High-frequency: Instabilities
  - Low-frequency: Self-organization

- **Multiphysics** nature
  - Collisional (intermolecular, walls)
  - Collisionless (non-Maxwellian, kinetic)

- **Power density**: \( P_d = j_D E = (j_i + j_e) E \)
  - Electric field: \( E \)
  - Electron current density: \( j_e = e n_e \mu_e E \)
  - Electron mobility: \( \mu_e \propto \begin{cases} B^{-2} & \text{(Classical)} \\ B^{-1} & \text{(Anomalous)} \end{cases} \)
Multiscale phenomena in partially magnetized plasmas

- Electromagnetic fields (typically static-B field is assumed)

**GHz (ns)**
- Electron transport (gyrofrequency)
  \[ \Omega = \frac{\omega_B}{\nu_m} \]
- Electron kinetics (collisions)

**MHz (µs)**
- Ion transport (acceleration, ionization)

**kHz (ms)**
- Neutral atom transport (gas flow)

**Frequency (Time scale)**

**Small-scale physics**
- Waves and instabilities
- Plasma-wall interaction
- Anomalous resistivity

**Device-scale phenomena**
- Breathing mode
- Azimuthally rotating spokes

**Extremely challenging to obtain reliable dynamic experimental data at >1-10 MHz (for now)**

- Laser Thomson scattering (plasma waves)
- Fast camera (optical)
- High-speed probe system
- Laser induced fluorescence

Next decade!

Significant progress in past decade

Validation

[Hara, PSST 28, 044001 (2019)]
Low-frequency (10-30 kHz) plasma oscillations

Azimuthally rotating spokes [Ellison 2011]

Mode transition: ionization oscillation [Hara 2014]

Breathing mode: high-speed probe [Lobbia PhD 2010]

Gradient drift instability [Kawashima & Hara 2018]
Physics-based modeling techniques for plasma flows

(a) Fluid (continuum) models
- Drift-diffusion model
- Euler/Navier-Stokes/MHD/Two-fluid
- Numerically inexpensive

(b) Particle-based kinetic methods
- 1 macroparticle ≈ $10^5$-$10^8$ real particles
- Particle-in-cell (PIC), DSMC, MCC

(c) Grid-based direct kinetic (DK) methods
- Solve kinetic equations directly in discretized phase space
- No statistical noise vs. particle method

Cf.) Knudsen number: $Kn = \lambda/L$
Plasma fluid modeling strategies

Drift-diffusion (DD) flux models

- **Quasineutral \((n_i \approx n_e)\) DD model:**
  \[
  \nabla \cdot (\vec{\Gamma}_i - \vec{\Gamma}_e) = 0 \quad \Rightarrow \quad \nabla \cdot (n_e \vec{\mu}_e \cdot \nabla \phi) = f(p_e, \vec{\Gamma}_i)
  \]
  [HPHall (Fife, Martinez-Sanchez, 1998), HPHall-2/3 (Ahedo 2006, Hofer 2008), Hall2De (Mikellides 2009), Detailed fluid model (Choi and Boyd 2008), other models (Boeuf and Garrigues 1998, Barral 2003, Hara 2014)]
  \[
  \frac{\partial n_e}{\partial t} + \nabla \cdot [\vec{\mu}_e \cdot (n_e \vec{E} + \nabla p_e)] = \dot{n}_e
  \]
  \[
  \nabla^2 \phi = -\frac{e}{\varepsilon_0} (n_i - n_e)
  \]

- **Non-neutral \((n_i \neq n_e)\) DD model:**
  Scharfetter-Gummel scheme, Dielectric relaxation time
  Used in LTP models [Kushner 2009]

Full fluid moment (FFM) model

Need non-oscillatory schemes for the nonlinear inertia term (cf. hyperbolic PDE)
Used in other fields, e.g. CFD and HTP [Hakim, Hammett, Srinivasan, Shumlak]

\[
\frac{\partial U}{\partial t} + \nabla \cdot \vec{F} = S, \text{ where } U = [n, n\vec{u}, n\varepsilon] \quad \& \quad \nabla^2 \phi = -\frac{e}{\varepsilon_0} (n_i - n_e)
\]
Fluid model: Comparison of DD vs FFM models

- Non-oscillatory results are obtained from all solvers (Quasineutral drift-diffusion [QDD], Non-neutral drift-diffusion [NDD], and full fluid moment models [FFM-T] [FFM-E]).
- Effects of the nonlinear inertia term (e.g. shear) on electron transport are observed: shear diamagnetic drift.

Number density

Effective Hall parameter

\[ \Omega_{eff} = \frac{nu_y}{nu_x} \]

“Anomalous" cross-field electron transport

**Classical theory:** electrons are trapped by the magnetic field

\[
\frac{d\vec{x}}{dt} = \vec{v}; \quad \frac{d\vec{v}}{dt} = \frac{q}{m} (\vec{E} + \vec{v} \times \vec{B})
\]

What if there is a plasma wave (i.e. fluctuation in the electric field)?

**Turbulence:** *Electron transport enhanced by plasma wave (1D, 2D, 3D?)*
2D z-theta benchmark testcase

- **Test case proposed by Boeuf and Garrigues 2018**
- **BC**: anode + electron injection
- **Fixed ionization rate**: run time \( \sim 30 \, \mu s \) (If neutrals are resolved, 1 ms is needed)

- 500x256 cells
- \( \sim 250 \) ppc
- 200 V
- 0.01 T peak
- \( j_i = 400 \, A/m^2 \)
- Xe\(^+\)
- 200 CPUs
- 2 weeks

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**Diagram**: Axial and Azimuthal drifts, ionization and periodicity. 
**Parameters**: 
- Anode (200 V)
- Ionization rates: \( \Gamma_{ea}, \Gamma_{ia} \)
- Magnetic field maximum
- Xe\(^+\) ions
- Emission plane: \( \Gamma_{ec} = \Gamma_{ec1} + \Gamma_{ec2} = \Gamma_{a} = \Gamma_{ea} - \Gamma_{ia} \)
- E-field magnitude and direction
- X-axis: \( L_x = 2.5 \, \text{cm} \)
- Y-axis: \( L_y = 1.28 \, \text{cm} \)
- \( x_{B_{\text{max}}} = 0.75 \, \text{cm} \)
Azimuthal plasma wave initiated by electron cyclotron drift instability (ECDI)

- ExB drift in the y-direction causes the plasma wave formation in the y-direction.
- Upstream: small scale fluctuation (dominant mode: ~1 mm)
- Fluctuating $E_y$ perturbs and detraps the electrons from the magnetic field lines

- Singly charge ions: Xe$^+$
- Plasma wave (1D in y)

$B_z \times E_x$

Drift $\vec{U} = \vec{E} \times \vec{B}$

Azimuthal electric field, $E_y$
Experimental evidence of axial (cross-field) plasma wave due to ion-ion two-stream instability (IITSI)

- Coherent Thomson scattering (CTS) detected signature of unambiguous axial plasma wave in the plume of a cross-field discharge (with azimuthal wave).
- Phase velocity of axial wave ($v_\phi$) is faster than Xe$^+$ ion velocity ($v_{i0}$)

$x \in [11, 30] \text{ mm}$

From the CTS measurement, it was observed that the stream of singly-charged ions (Xe\(^+\)) and doubly-charged ions (Xe\(^{2+}\)) causes ion-ion two-stream instability (IITSI).

Doubly charged ion fraction: \( \alpha = \frac{2n_i^{2+}}{n_e} \)

\[
(k \parallel \lambda_D)^2 \left[ 1 - \frac{(1 - \alpha)\omega_{pi}^2}{(\omega - k \cdot U_i^+)^2} - \frac{\alpha\omega_{pi}^2}{(\omega - k \cdot U_i^{2+})^2} \right] + 1 - I_0(b) \exp(-b) + \sum_{n=1}^{\infty} \frac{2\omega^2 I_n(b) \exp(-b)}{(n\omega_B)^2 - \omega^2} = 0
\]

(a) ECDI only

(b) IITSI only

(c) IITSI+ECDI
Multidimensional plasma microturbulence

Multidimensional (2D) plasma wave is observed due to the coupling of (i) **axial** oscillation via IITSI and (ii) **azimuthal** oscillation via ECDI

100% Xe$^+$
(ECDI only)

80% Xe$^+$, 20% Xe$^{2+}$
(ECDI+IITSI)
Cross-field transport: electrons and ions

Nonlinear coupling of ECDI and IITSI

\[ |u_{xe}| \approx \mu E_x \]
increases with more Xe\(^{2+}\)

Deceleration of doubly charged ions

Acceleration of singly charged ions
Time-averaged electron streamline

Fluctuation-based electron transport
[Waltz PoF 1982, Liewer Nucl Fusion 1985]

\[ \langle \Gamma_{ex} \rangle = \frac{n'_c E'_y}{B_z} \]
\[ \langle \Gamma_{ey} \rangle = -\frac{n_e x_0 E_{x0}}{B_z} - \frac{n'_c E'_x}{B_z} \]

100% Xe$^+$ (ECDI only) \[ |\langle \Gamma_{ex} \rangle| < |\langle \Gamma_{ey} \rangle| \]

80% Xe$^+$, 20% Xe$^{2+}$ (ECDI+IITSI) \[ |\langle \Gamma_{ex} \rangle| > |\langle \Gamma_{ey} \rangle| \]

Color: amplitude of electric field. Line: electron streamlines.
Broadening (heating) of the ion VDFs is observed: saturation of IITSI

100% Xe$^+$ (ECDI only)

80% Xe$^+$, 20% Xe$^{2+}$ (ECDI+IITSI)
Summary: anomalous transport

- We investigated the coupling of ECDI (azimuthal) and IITSI (axial) modes using a 2D parallel PIC code. The numerical results are consistent with the theoretical analysis and experimental observation (cf. coherent Thomson scattering).

- The enhancement of cross-field electron transport is observed due to the excitation of multidimensional (2D) plasma turbulence via the nonlinear saturation of the linear instabilities.

Thank you for your attention

Plasma Dynamics Modeling Laboratory (PDML)  
Aeronautics and Astronautics, Stanford University  
Webpage: pdml.stanford.edu  
Email address: kenhara@stanford.edu

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