Uncertainty Quantification and Credible Predictions for Reduced-Fidelity Modeling of Porous Electrospray

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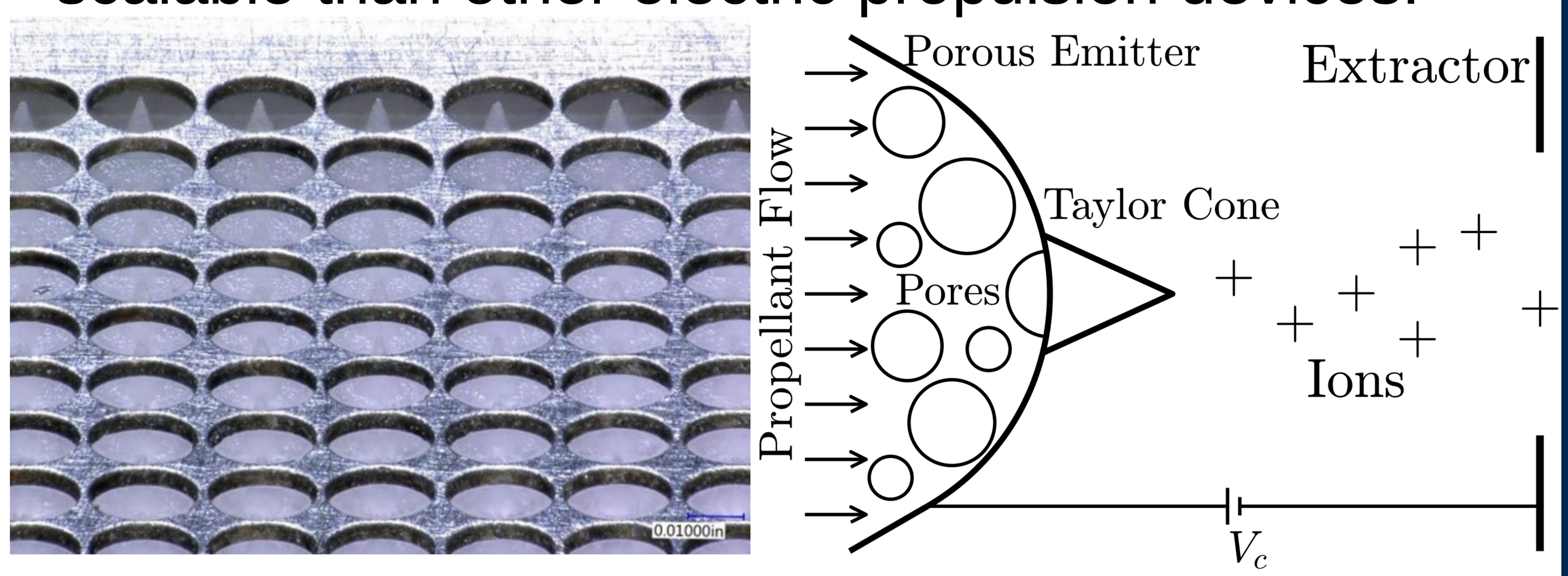
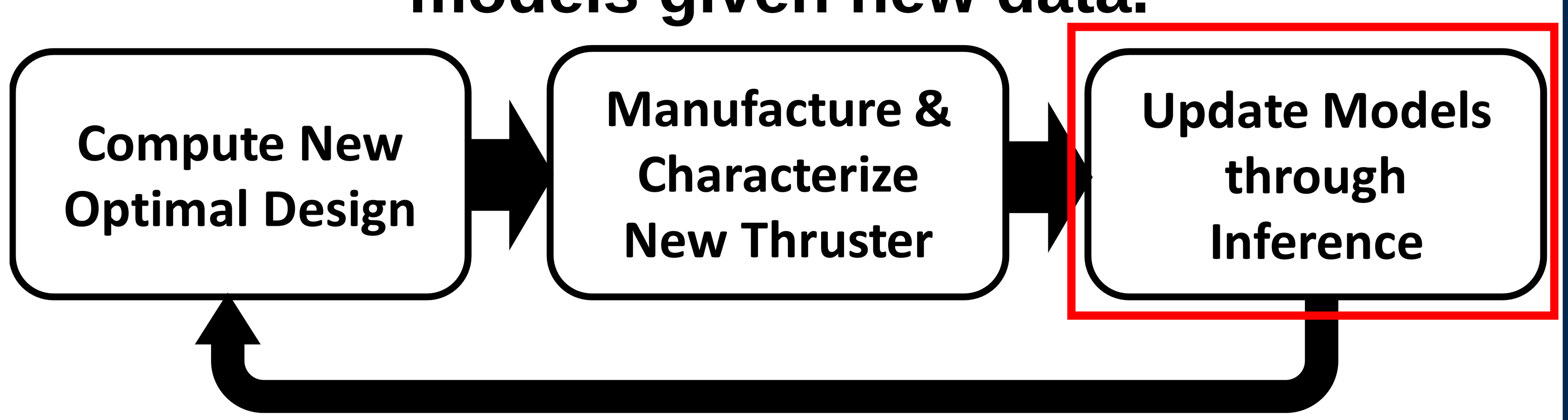
Motivation

Electrospray thrusters are electrohydrodynamic devices that extract charged particles from a conductive liquid to produce thrust in space. In principle, this process is more efficient and scalable than other electric propulsion devices.



But electrosprays are small, requiring arrays of 10^2-10^4 individual emitters. At this scale, lifetime and performance can be limited by physical and manufacturing uncertainty. We aim to address this problem with a robust design optimization and testing loop.

We focus here on updating reduced-fidelity models given new data.



Objectives

Reduced-fidelity modeling engenders physical uncertainty by simplifying physics.

1. How do we rigorously account for this uncertainty?
2. Can we use data to update our state of knowledge?
3. How can we use our knowledge to credibly predict device performance?

Methodology

Bayes' Theorem:

$$\text{prob}(\theta|X,I) = \frac{\text{prob}(X|\theta,I) \times \text{prob}(\theta|I)}{\text{prob}(X|I)}$$

θ: model parameters
X: data
I : background knowledge

Suppose we have a data set, X

⇩

Propose a simplified model:
$y = mx + b$

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Model uncertain from experimental error and simplification

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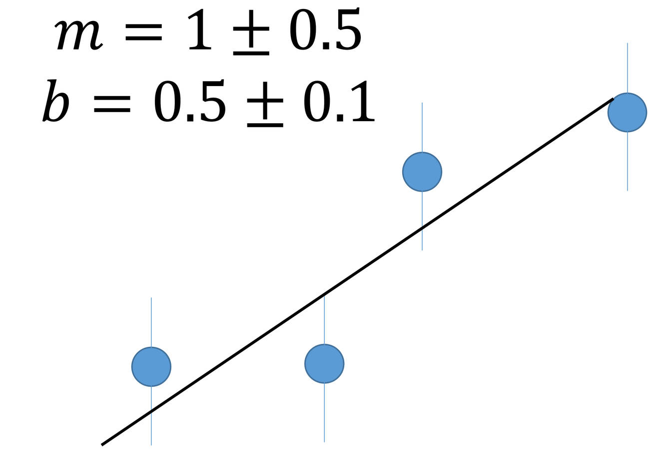
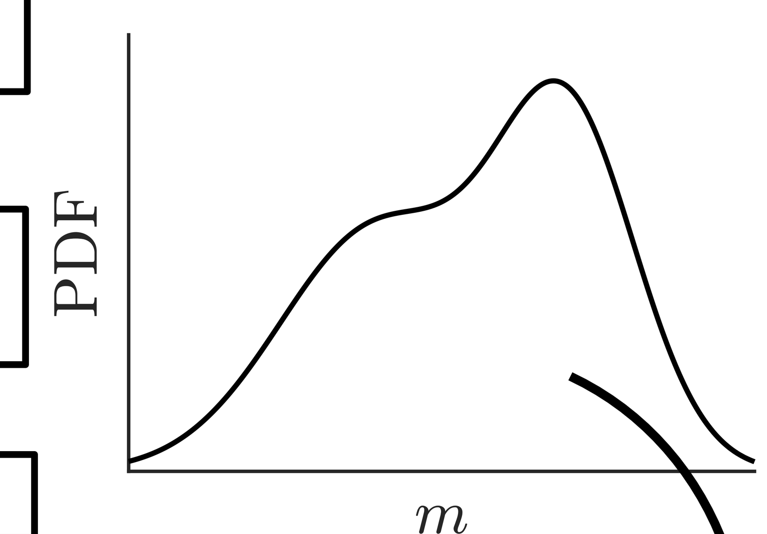
Posterior distributions over parameters

$m = 1 \pm 0.5$
$b = 0.5 \pm 0.1$





Use existing modeling framework

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ESPET

ElectroSpray Propulsion Engineering Toolkit

Sample Many Times to Make Predictions

m, b m, b m, b m, b m, b m, b m, b

Models

$$I_{ion} = I_{ion,0} + \zeta \frac{V - V_0}{C_R}$$

$$I_{jet} = \alpha\, I_{jet,0} \left(\frac{Q}{Q_0}\right)^{1/2}$$

$$i = (N_{max} - 1)\left(1 - \frac{(r_{base} - r_0)}{b_0}\right)^4 + 1$$

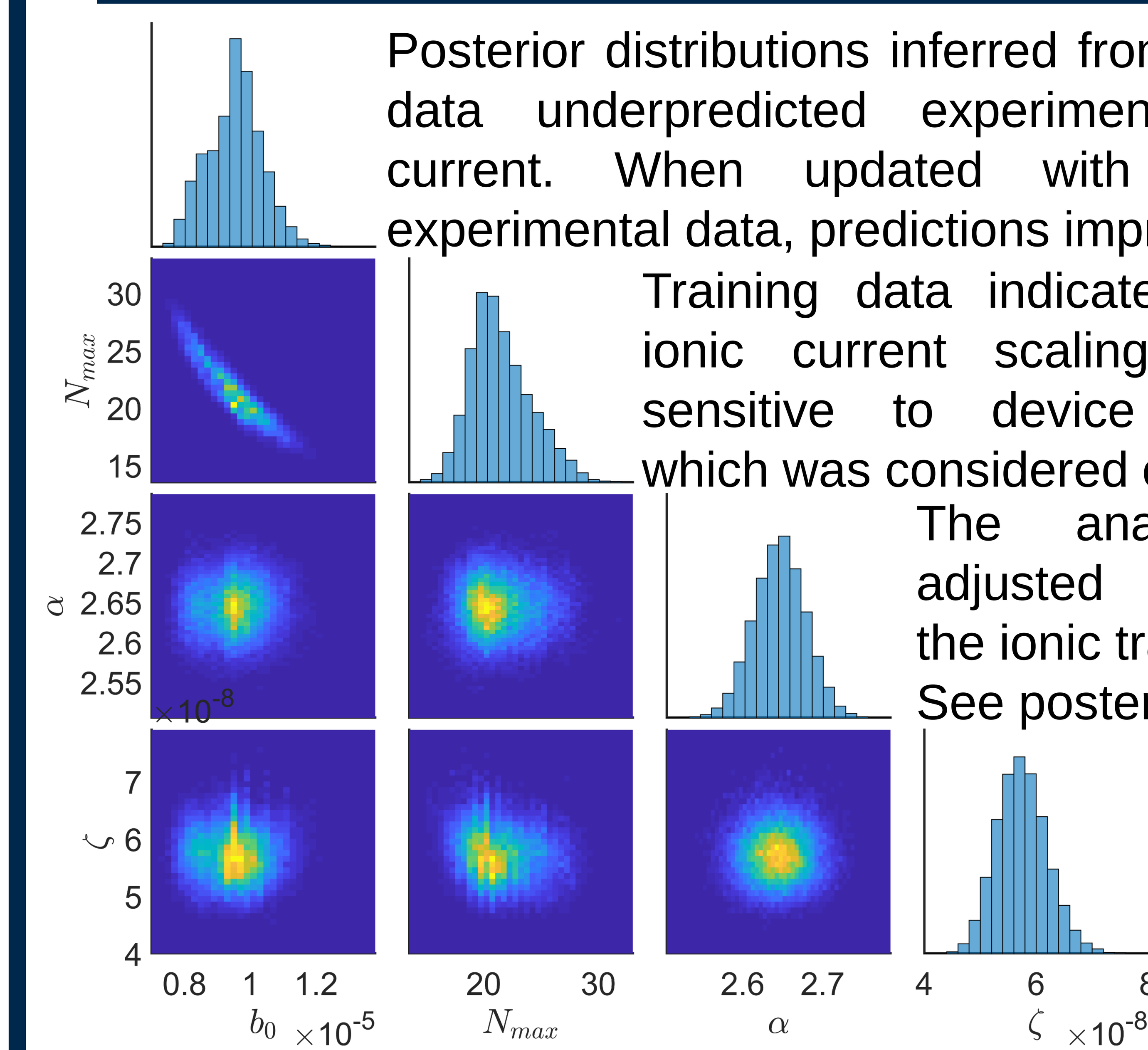
$$r_{base} = \frac{4\gamma a^2 \,\text{atanh}^2(\eta_0)\, \left(1 - \eta_0^2\right)^2}{\varepsilon_0 V^2}$$

Model Inputs:

- V: emitter voltage
- I: current (ionic or jet)
- Q: vol. flow rate
- r_0: pore radius
- γ: surface tension
- C_R: scale hyd. Imped.
- V_0: onset voltage
- I_0: onset/scale current
- Q_0: scale vol. flow rate
- a, η_0: emitter geometry

Inferred Parameters: $\zeta, \alpha, b_0, N_{max}$

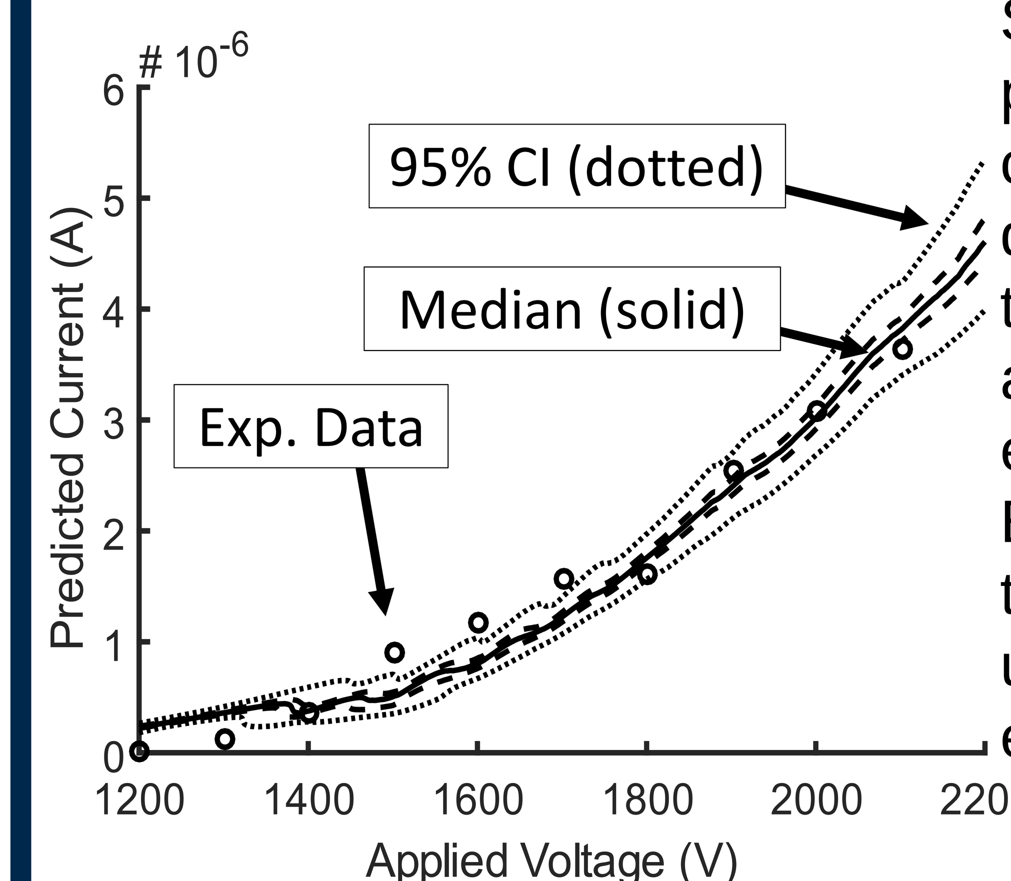
Results



Posterior distributions inferred from published data underpredicted experimental emitter current. When updated with the new experimental data, predictions improved.

Training data indicated that the ionic current scaling is highly sensitive to device geometry, which was considered certain.

The analysis was adjusted to exclude the ionic training data. See posteriors on left.



Sampling from the revised posterior distributions more closely matched empirical data (see left). Considering these uncertainties creates a prediction envelope that encodes our knowledge. Extending this framework to additional models and uncertain parameters could enable robust design.

Conclusions

1. Bayesian inference can be used to describe the physical uncertainty of reduced-fidelity models.
2. Performance is highly sensitive to uncertain device geometry, motivating a probabilistic problem.
3. Additional model development may be necessary to capture other phenomena.

References

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