Modeling of Turbulent Radiative Shocks with Applications to High Energy Density Physics and Astrophysics

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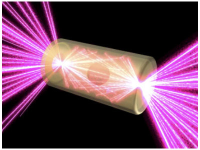
29 September 2010

This work was performed under the DOE NNSA Predictive Science Academic Alliances Program by grant DE-FC52-08NA28616 and under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344

Investigation models turbulent radiation hydrodynamics in the diffusion approximation and evaluates its effects on radiative blast waves

- Radiation transport, shock physics, and turbulence are intimately coupled in several HEDP and astrophysical environments
  - Supernovae, competing processes in stellar life cycles, black hole dynamics
  - Z-pinches, high-energy laser experiments

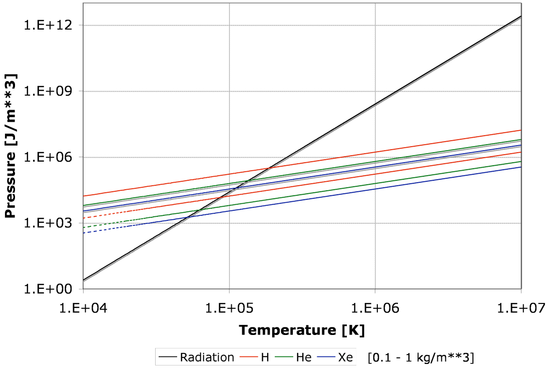




- Blast waves created by such phenomena are susceptible to instabilities
  - Rayleigh-Taylor, Kelvin-Helmholtz, Richtmyer-Meshkov
- Theoretical and experimental studies typically focus on one or two of the processes: radiation, shock physics, turbulence
- This study models radiation hydrodynamics via equilibrium diffusion and turbulence by a Reynolds-averaged Navier-Stokes (RANS) model

Radiation fields directly influence hydrodynamics in extreme temperature and pressure environments

- Radiation to hydrodynamic pressure ratio = $a_R T^4/(3\rho c_s^2)$



- Radiation quickly dominates system as temperature increases
  - $\frac{p_M}{p_R}(T = 10^6 K) \approx 5.95 \times 10^{-3}$, $\frac{p_M}{p_R}(T = 10^7 K) \approx 6.55 \times 10^{-6}$

Reynolds and Favre decompositions along with gradient-diffusion approximations are used to provide closed turbulent transport equations

- Reynolds and Favre averaging can be expressed as ordinary and density-weighted temporal means, respectively

$$\overline{\phi} = \lim_{\tau\to\infty} \frac{1}{\tau} \int_t^{t+\tau} \phi(\mathbf{x}, t)\, \mathrm{d}t \quad , \quad \widetilde{\varphi} = \frac{1}{\overline{\rho}} \lim_{T\to\infty} \frac{1}{T} \int_t^{t+T} \rho(\mathbf{x}, t)\, \varphi(\mathbf{x}, t)\, \mathrm{d}t$$

- Reynolds and Favre decompositions are given by:
  - $\rho = \overline{\rho} + \rho'$, $p = \overline{p} + p'$, $E_R = \overline{E}_R + E_R'$, $F_j = \overline{F}_j + F_j'$
  - $v_j = \widetilde{v}_j + v_j''$, $U = \widetilde{U} + U''$, $T = \widetilde{T} + T''$
- Averaging system of interest and using decompositions leads to fluctuating correlations closed via gradient-diffusion closures
- Gradient-diffusion approximation uses turbulent kinetic energy, K, and dissipation rate, ϵ, to form the turbulent viscosity needed for the closures

$$\nu_t = C_\mu \frac{K^2}{\epsilon} = \frac{\mu_t}{\overline{\rho}}$$

Turbulent radiative gas dynamics is achieved by Reynolds averaging equilibrium diffusion model and generalizing gradient-diffusion closures

- Given total energy $\overline{\rho}\,\widetilde{\mathcal{E}} = \overline{\rho}\left(\widetilde{v}^{\,2}/2 + \widetilde{U} + K\right) + \overline{E}_R$, total pressure $\overline{p}^* = \overline{p}_M + \overline{p}_R$, Reynolds stress tensor τ_{ij}, and radiative pressure dilatation Π^*, the first five model equations are

Mass
$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{\rho}\,\widetilde{v}_j\right) = 0$$

Momentum
$$\frac{\partial}{\partial t}\left(\overline{\rho}\,\widetilde{v}_i\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\,\widetilde{v}_i\,\widetilde{v}_j\right) = \overline{\rho}\,g_i - \frac{\partial \overline{p}^*}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j}$$

Total Energy
$$\frac{\partial}{\partial t}\left(\overline{\rho}\,\widetilde{\mathcal{E}}^*\right) + \frac{\partial}{\partial x_j}\left[\left(\overline{\rho}\,\widetilde{\mathcal{E}}^* + \overline{p}^*\right)\widetilde{v}_j\right] = \overline{\rho}\,g_i\,\widetilde{v}_i - \frac{\partial}{\partial x_j}\left[\frac{\left(\overline{p}^* + \overline{E}_R\right)\nu_t}{\sigma_\rho\,\overline{\rho}}\,\frac{\partial \overline{\rho}}{\partial x_j}\right]$$
$$- \frac{\partial}{\partial x_j}\left(\tau_{ij}\,\widetilde{v}_i\right) + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_U}\,\frac{\partial \widetilde{U}}{\partial x_j} + \frac{\mu_t}{\sigma_K}\,\frac{\partial K}{\partial x_j}\right) + \frac{\partial}{\partial x_j}\left(\frac{4}{3}\,\frac{\nu_t}{\sigma_{E_R}}\,\frac{\partial \overline{E}_R}{\partial x_j}\right) - \frac{\partial \overline{F}_j^{\,n}}{\partial x_j}$$

Turbulent K Energy
$$\frac{\partial}{\partial t}\left(\overline{\rho}\,K\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\,K\,\widetilde{v}_j\right) = -\frac{\nu_t}{\sigma_\rho\,\overline{\rho}}\,\frac{\partial \overline{\rho}}{\partial x_j}\,\frac{\partial \overline{p}^*}{\partial x_j} - \tau_{ij}\,\frac{\partial \widetilde{v}_i}{\partial x_j} - \overline{\rho}\,\epsilon + \Pi^* + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_K}\,\frac{\partial K}{\partial x_j}\right)$$

Dissipation Rate
$$\frac{\partial}{\partial t}\left(\overline{\rho}\,\epsilon\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\,\epsilon\,\widetilde{v}_j\right) = -\frac{\epsilon}{K}\left[C_{\epsilon 0}\,\frac{\nu_t}{\sigma_\rho\,\overline{\rho}}\,\frac{\partial \overline{\rho}}{\partial x_j}\,\frac{\partial \overline{p}^*}{\partial x_j} + C_{\epsilon 1}\,\tau_{ij}\,\frac{\partial \widetilde{v}_i}{\partial x_j} + C_{\epsilon 2}\,\overline{\rho}\,\epsilon - C_{\epsilon 3}\,\Pi^*\right]$$
$$+ \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_\epsilon}\,\frac{\partial \epsilon}{\partial x_j}\right)$$

Turbulence contributions introduced via mean radiative flux introduce need for transport equations for density and temperature variances

- Classical and mean radiative fluxes with opacity model $\Sigma_A(\rho, T)$ for $1 \leq n \leq 3$ are

$$F_j^{\,n} = \frac{c}{3\,\Sigma_A(\rho,\,T)}\,\frac{\partial\,E_R}{\partial x_j} \quad , \quad \Sigma_A(\rho,\,T) = \beta\,\frac{\rho}{T^n}$$

$$\overline{F}_j^{\,n=1} = -\frac{a_R\,c\,\widetilde{T}}{3\,\beta\,\overline{\rho}}\left[\left(1 + 2\,C_T\,\frac{\sqrt{\overline{\rho'^2}\,\widetilde{T''^2}}}{\overline{\rho}\,\widetilde{T}} + \frac{\overline{\rho'^{\,2}}}{\overline{\rho}^{\,2}}\right)\frac{\partial\widetilde{T^4}}{\partial x_j} - \frac{2}{\lambda_\rho\,\overline{\rho}}\sqrt{\frac{2\,\overline{\rho'^2}}{\pi}}\,\widetilde{T^4}\right]$$

$$-\frac{a_R\,c}{3\,\beta\,\overline{\rho}}\left[\frac{\partial}{\partial x_j}\left(\widetilde{T^5} - \widetilde{T}\,\widetilde{T^4} - C_T\,\frac{\sqrt{\overline{\rho'^2}\,\widetilde{T''^2}}}{\overline{\rho}}\,\widetilde{T^4}\right) - \frac{2}{\lambda_T}\left(\widetilde{T^5} - \widetilde{T^4}\,\widetilde{T}\right) + C_T\,\widetilde{T^4}\,\frac{\partial}{\partial x_j}\left(\frac{\sqrt{\overline{\rho'^2}\,\widetilde{T''^2}}}{\overline{\rho}}\right)\right]$$

- Compressible turbulent and PDF closures are used in density variance, $\overline{\rho'^2}$, and temperature variance, $\widetilde{T''^{\,2}}$, transport equations development

$$\text{Dens.Var.} \quad \frac{\partial\overline{\rho'^2}}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{\rho'^2}\,\widetilde{v}_j\right) = \frac{2\,\nu_t}{\sigma_\rho}\left(\frac{\partial\overline{\rho}}{\partial x_j}\right)^2 - \overline{\rho'^2}\,\frac{\partial\widetilde{v}_j}{\partial x_j} - C_{\rho^2}\,\frac{\epsilon}{K}\,\overline{\rho'^2} - \frac{2\,\overline{\rho}^{\,2}\,\Pi^*}{\gamma\,\overline{p}} + \frac{\partial}{\partial x_j}\left(\frac{\nu_t}{\sigma_{\rho^2}}\,\frac{\partial\overline{\rho'^2}}{\partial x_j}\right)$$

$$\text{Temp.Var.} \quad \frac{\partial}{\partial t}\left(\overline{\rho}\,\widetilde{T''^{\,2}}\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\,\widetilde{T''^{\,2}}\,\widetilde{v}_j\right) = 2\left[\frac{\mu_t}{\sigma_U}\left(\frac{\partial\,\widetilde{T}}{\partial x_j}\right)^2 - (\gamma - 1)\,\overline{\rho}\,\widetilde{T''^{\,2}}\,\frac{\partial\widetilde{v}_j}{\partial x_j}\right]$$

$$-2\,(\gamma - 1)\,C_T\,\frac{\epsilon}{K}\,\widetilde{T}\,\sqrt{\overline{\rho'^{\,2}}\,\widetilde{T''^{\,2}}} + \frac{\partial}{\partial x_j}\left(\frac{\mu_t}{\sigma_U}\,\frac{\partial\,\widetilde{T''^{\,2}}}{\partial x_j}\right)$$

Rankine-Hugoniot jump relations ensure total mass, momentum, and energy conservation and provide post shock relations

- Exact relations for profiles behind strong turbulent-radiative shocks as functions of shock speed, $\widetilde{v}_s$, are given by

$$\text{Density}: \quad \frac{\overline{\rho}_2}{\overline{\rho}_1} = \frac{(\gamma+1)\,\overline{p}_2^* + (\gamma-1)\,\tau_2 + 2(4-3\,\gamma)\,\overline{p}_{R,2}}{(\gamma-1)\,\overline{p}_2^*}\left[1+\frac{2}{\overline{p}_2^*}\left(\frac{\tau_2}{2} - \overline{\rho}_1\,K_2 - \frac{\Gamma_2^*}{\widetilde{v}_1}\right)\right]^{-1}$$

$$\text{Velocity}: \quad \widetilde{v}_2 = \widetilde{v}_s\left\{1 - \frac{(\gamma-1)\,\overline{p}_2^*}{(\gamma+1)\,\overline{p}_2^* + (\gamma-1)\,\tau_2 + 2(4-3\,\gamma)\,\overline{p}_{R,2}}\left[1+\frac{2}{\overline{p}_2^*}\left(\frac{\tau_2}{2} - \overline{\rho}_1\,K_2 - \frac{\Gamma_2^*}{\widetilde{v}_s}\right)\right]\right\}$$

$$\text{Pressure}: \quad \overline{p}_2^* = \overline{\rho}_1\,\widetilde{v}_s^{\,2}\left\{1 - \frac{(\gamma-1)\,\overline{p}_2^*}{(\gamma+1)\,\overline{p}_2^* + (\gamma-1)\,\tau_2 + 2(4-3\,\gamma)\,\overline{p}_{R,2}}\left[1+\frac{2}{\overline{p}_2^*}\left(\frac{\tau_2}{2} - \overline{\rho}_1\,K_2 - \frac{\Gamma_2^*}{\widetilde{v}_s}\right)\right]\right.$$

$$-\tau_2$$

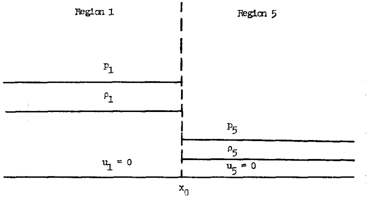
$$\Gamma^* = \frac{(\overline{p}^* + 3\,\overline{p}_R)\,\nu_t}{\sigma_\rho\,\overline{\rho}}\frac{\partial\,\overline{\rho}}{\partial x_j} - \frac{\mu_t}{\sigma_U}\frac{\partial \widetilde{U}}{\partial x_j} - \frac{\mu_t}{\sigma_K}\frac{\partial K}{\partial x_j} - \frac{4\,\nu_t}{\sigma_{E_R}}\frac{\partial \overline{p}^*}{\partial x_j} + \overline{F}^{\,n}_{j,F}$$

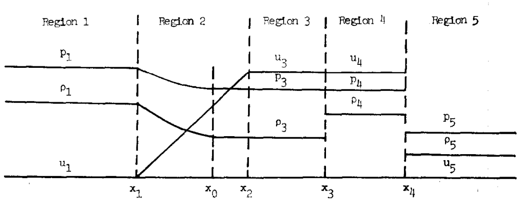
- As a check, results for a strong classical shock are obtained when removing turbulence and radiative effects

$$\frac{\rho_2}{\rho_1} = \frac{\gamma+1}{\gamma-1}\quad,\quad v_2 = \frac{2\,v_s}{\gamma+1}\quad,\quad p_2 = \frac{2\,\rho_1 v_s^2}{\gamma+1}$$

Weighted Essentially Non Oscillatory (WENO) and Riemann solvers will be used to simulate proposed turbulent radiation hydrodynamics model

- Sod reference problem is used to test early stage computational work
- Problem depicts two regions ($\gamma = 1.4$) under conditions:
  - $\rho_1 = 1.0$, $p_1 = 1.0$, $v_1 = 0$ || $\rho_5 = 0.125$, $p_5 = 0.10$, $v_5 = 0$





Ongoing and future work, and special thanks

- Physics of underlying turbulent radiative shocks has been investigated
  - An equilibrium diffusion model describes radiation hydrodynamics
  - A four-equation Reynolds-averaged Navier-Stokes (RANS) model is used to describe turbulence effects
  - Gradient-diffusion and similarity closures were generalized to account for radiative effects
- WENO methods and approximate Riemann solvers will be used for conducting numerical investigations
- Particular interest lies in studying these processes in planar, cylindrical, and spherical geometries for applications relevant to supernovae, black hole dynamics, high-energy laser experiments, and Z-pinches
- Propose experiments that can be used to verify this model

- Special Thanks
  - Professors William R. Martin, Kenneth Powell, John Foster
  - Rhys Ulerich (University of Texas), Jinan Yang, Ed Birdsall, S. Lee