Zero-dimensional modeling limitations for the Hall thruster breathing mode

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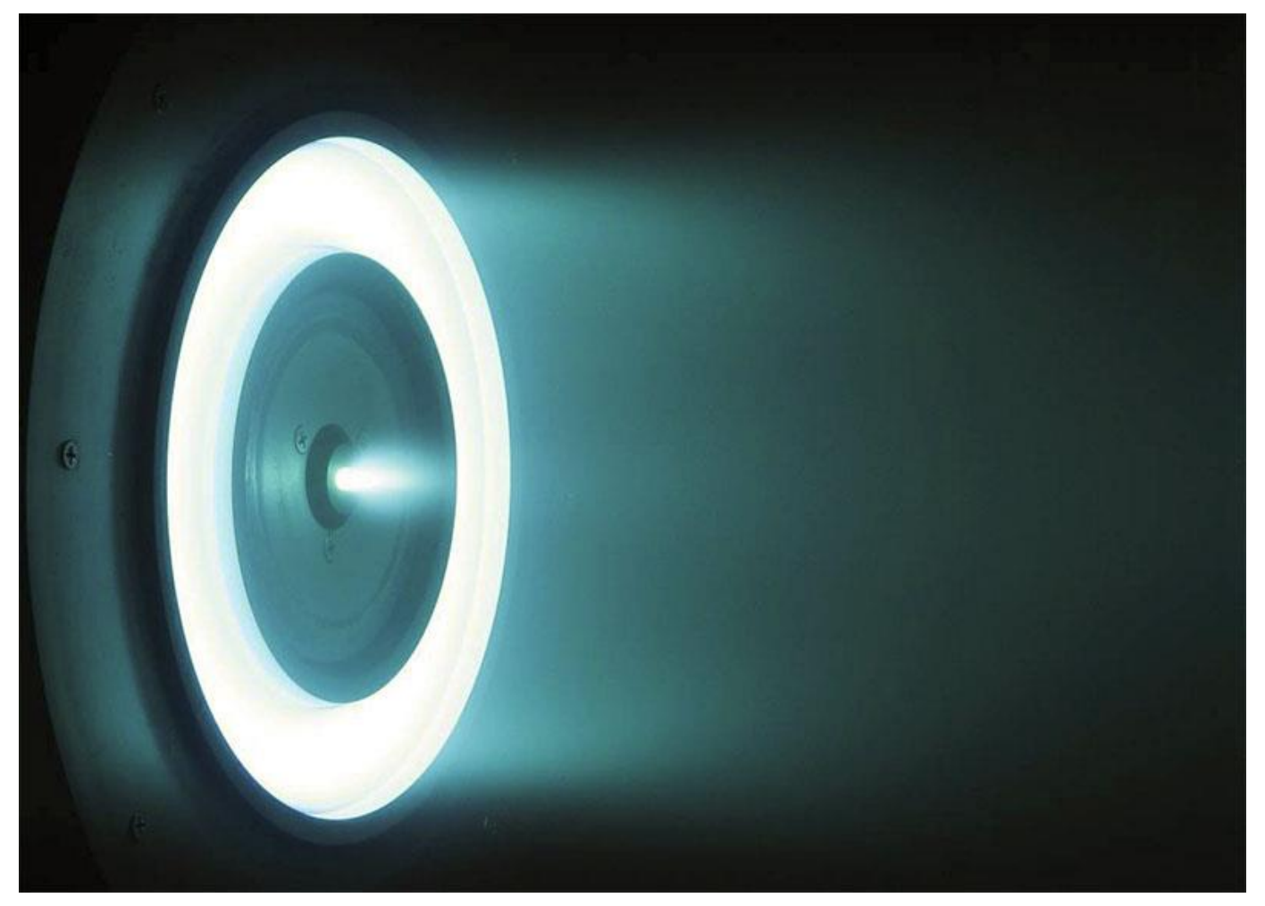
Background

Hall thrusters are a type of electric space propulsion that is increasingly used for Earth orbit and deep space missions. A Hall thruster uses crossed electric and magnetic fields to sustain a plasma and accelerate ions out of the device, producing thrust. Hall thrusters are very efficient, with exit velocities ~10 km/s for xenon propellant, but produce low thrust, ~100 mN. Although these devices have been studied and flown extensively, there are no self-consistent simulations of them. One phenomenon that is not completely understood is the ubiquitous "breathing mode": strong ~10 kHz oscillations in discharge current and other global parameters.



The breathing mode has been recovered by simulation [1] and characterized experimentally [2], but analytical approaches have yielded neither a intuitive explanation of the underlying physical mechanisms nor any criteria for instability [3].

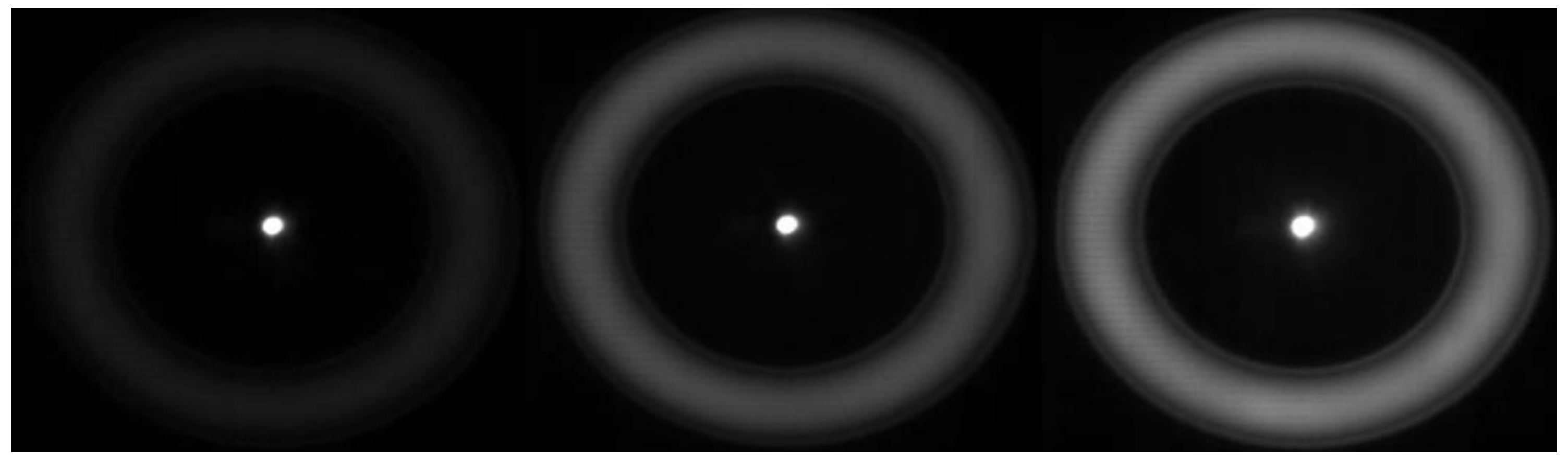


Figure 1: The H6 Hall thruster operating nominally (top), and a progression of still images of the channel during breathing (bottom), where each still is separated by ~10 μs.

Objectives

The breathing mode is sensitive to many operating parameters and may have large-scale effects on the thruster operation, for instance by playing a role in thruster erosion. Yet there are still many shortcomings with the modern understanding:

- Simulation: not predictive, not validated
- Experiment: impractical, unrealistic
- Theory: no intuitive criteria

- **What is the energy source?**
- **What are the instability criteria?**

Governing Equations

ion continuity
$$\frac{dn}{dt} = \xi_{iz} n n_n - \frac{u_i n}{L_{ch}} - \frac{2 u_w n}{R}$$

neutral continuity
$$\frac{dn_n}{dt} = -\xi_{iz} n n_n - \frac{u_n n_n}{L_{ch}} + \frac{u_n n_{int}}{L_{ch}}$$

ion momentum
$$\frac{dnu_i}{dt} = \frac{e}{m_i} nE - \frac{{u_i}^2 n}{L_{iz}}$$

electron energy
$$\frac{d}{dt}\left(\frac{3}{2} n T_e\right) = -\frac{5}{2}\frac{n T_e u_e}{L_{iz}} - n u_e E - n \epsilon_w \nu_w - n n_n \xi_{iz} \epsilon_{iz} \chi$$

Ohm's law
$$E = \eta \Omega^2 j = \frac{-e u_e B_r^2}{\nu(\alpha) m_e}$$

ionization length
$$\frac{dL_{iz}}{dt} = -u_f = n \xi \ell - u_n$$

Results

The stability of different subsets of the governing equations is judged by performing numerical (nonlinear) simulations and a linear perturbation analysis. A positive linear growth rate γ is desired.

Table 1: Combinations of perturbed quantities explored for 0D modeling.

		I	II	III	IV	V
Perturbed Quantity	n	X	X	X	X	X
	n_n	X	X	X	X	X
	u_i		X	X	X	
	T_e			X	X	
	E				X	
	L_{iz}					X

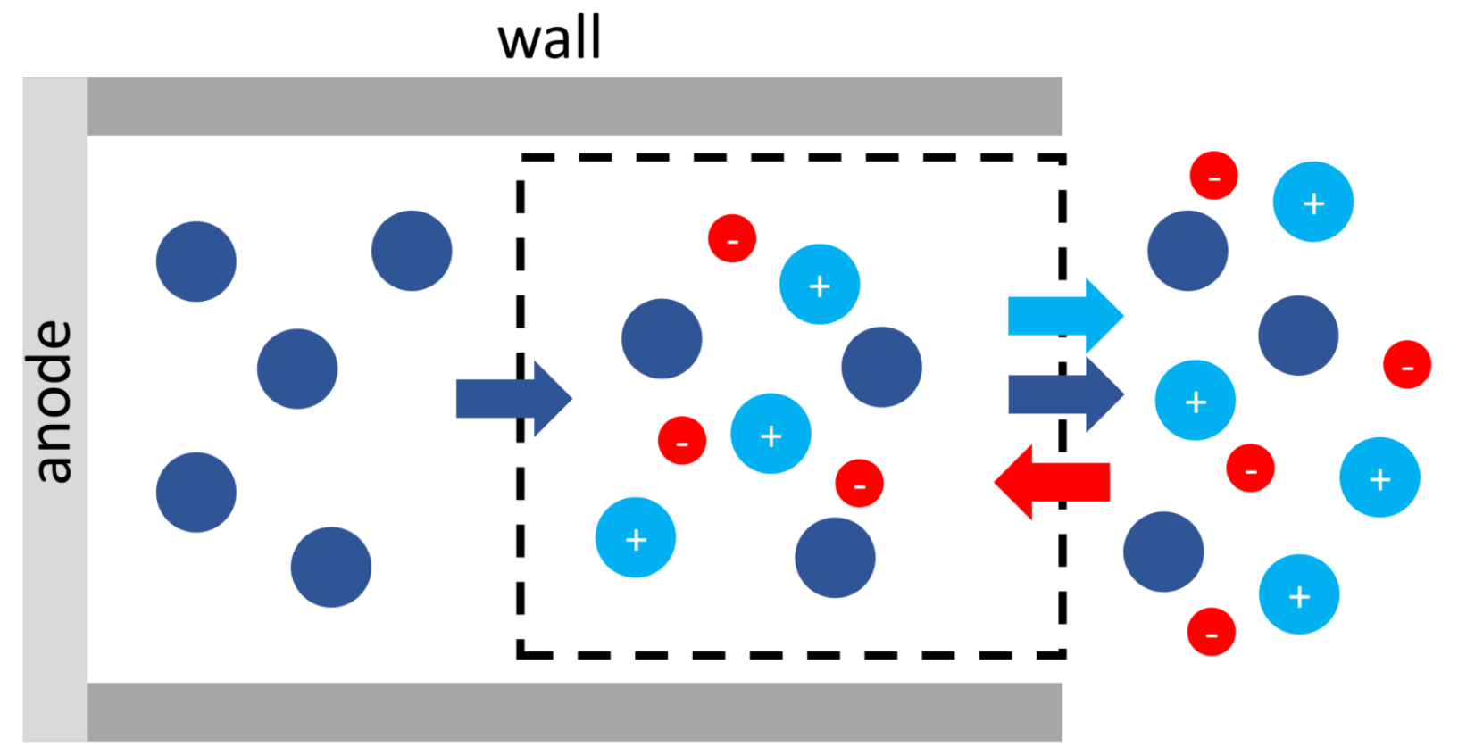


Figure 2: Physical interpretation of a 0D breathing model.

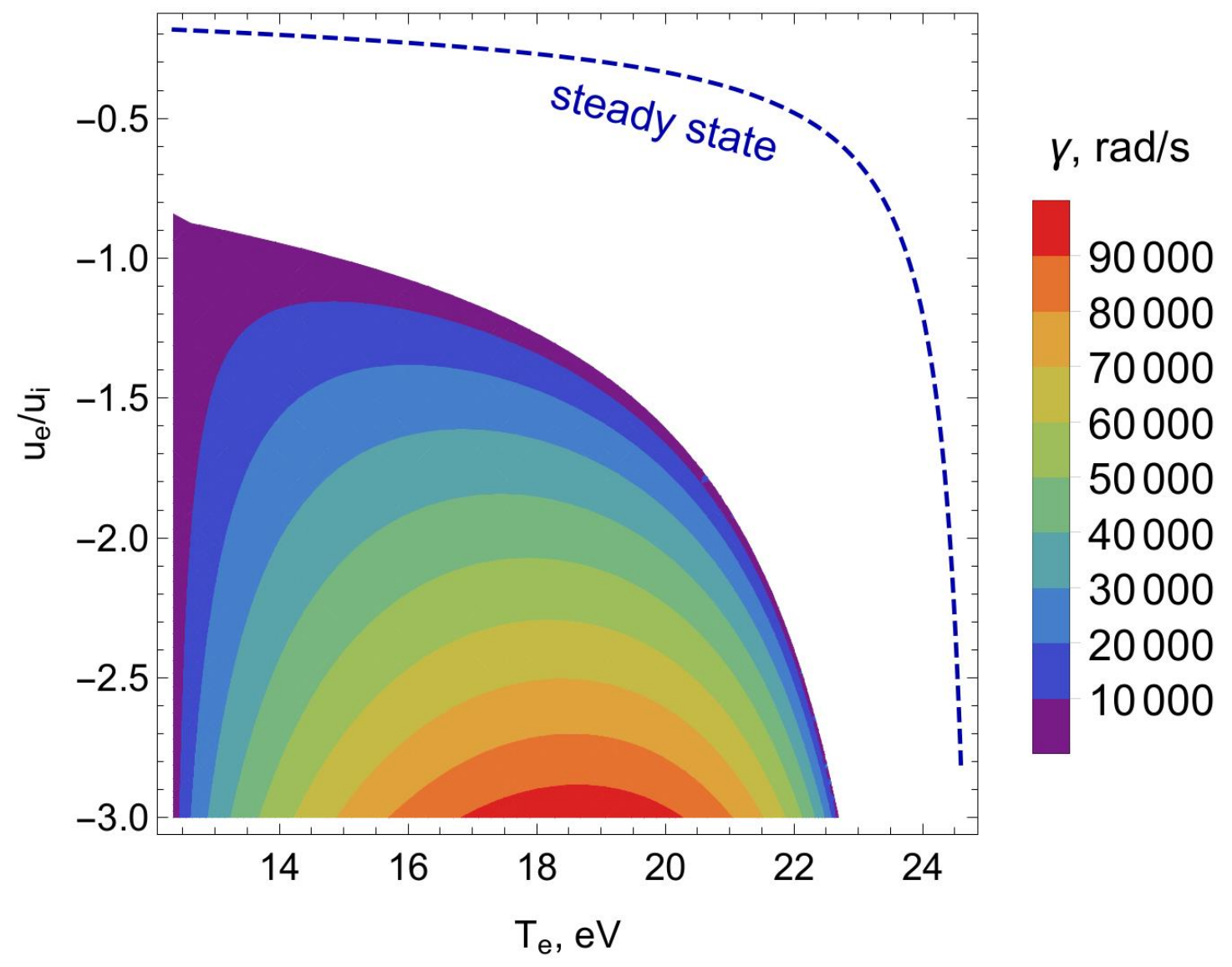


Figure 3: Linear stability for Case III.

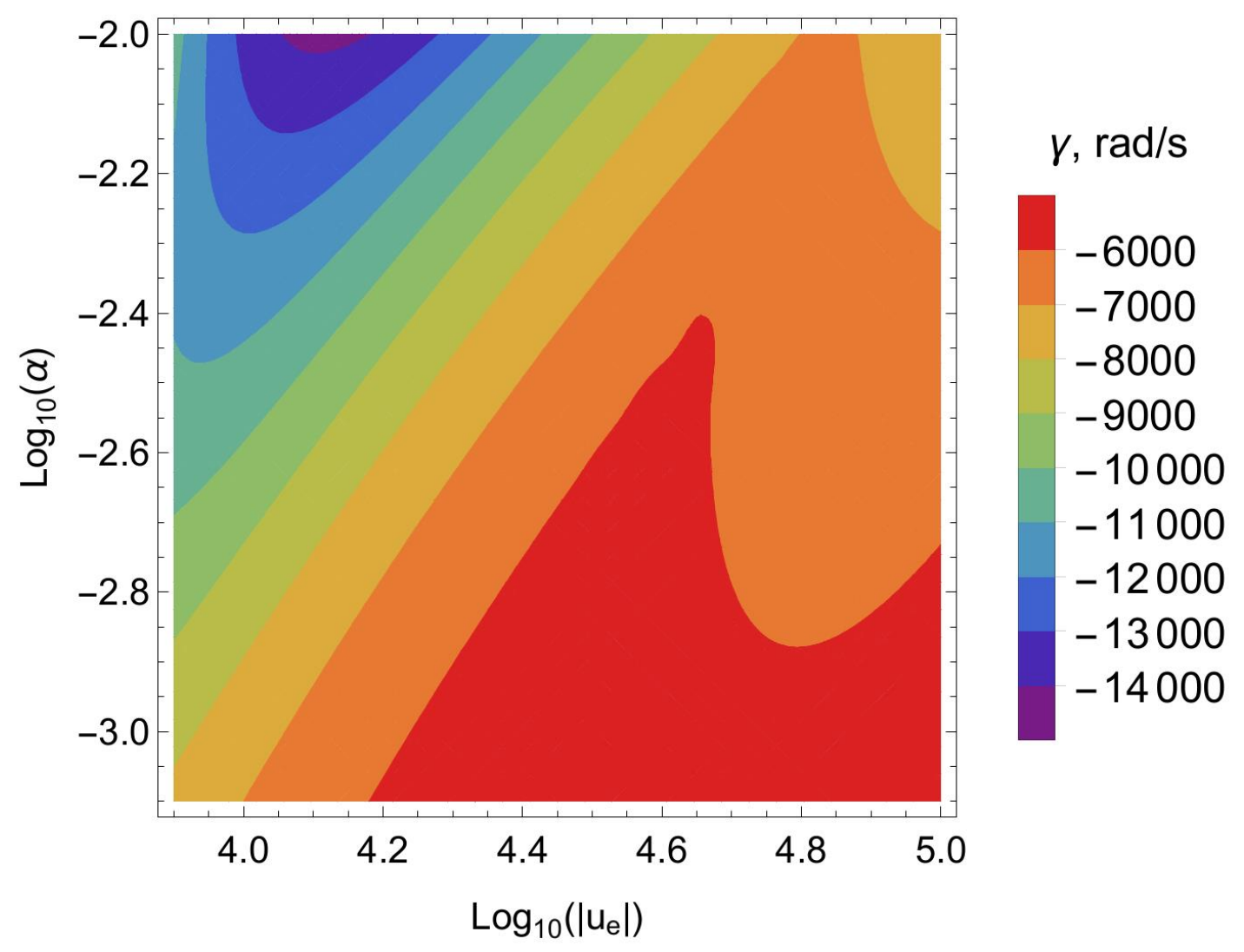


Figure 4: Linear stability for Case IV.

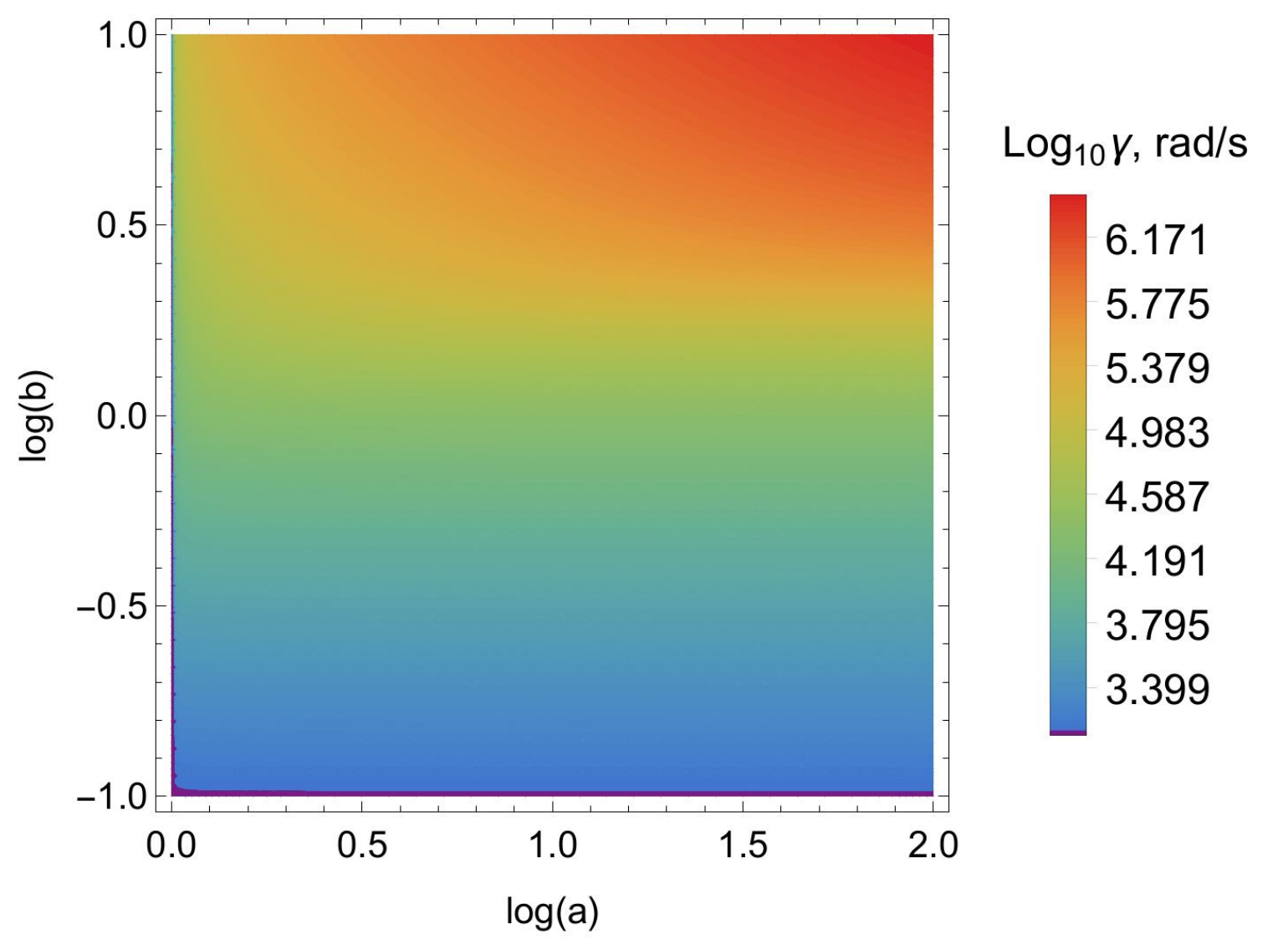


Figure 5: Linear stability for Case V.

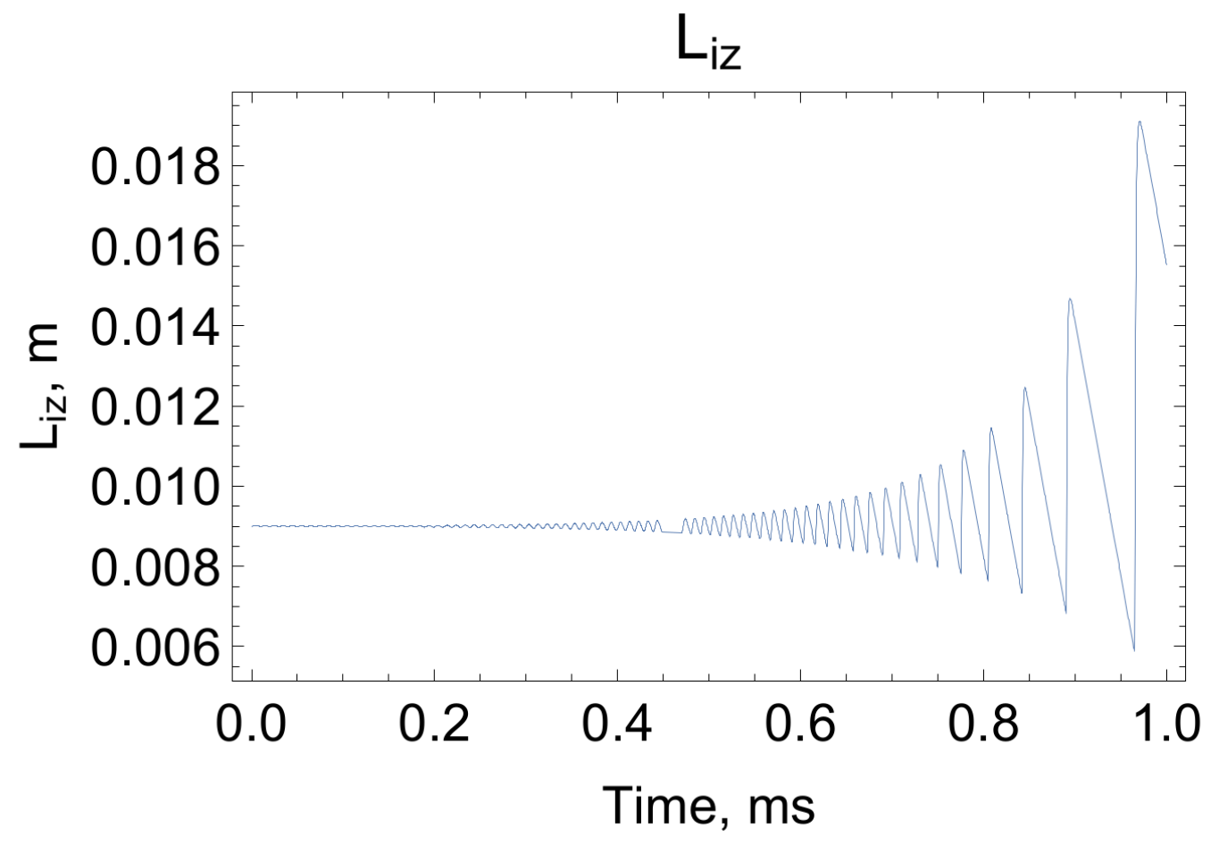


Figure 6: Time response of L_{iz} for Case V.

$$\mathrm{Re}(\omega) = 0$$

$$\gamma \approx \frac{u_i}{2L_{iz}}$$

Adding ionization region length perturbations is sufficient to yield instability. However, exact treatment of this model can only be done for a simplified system, and even then the instability is unconditional and the real frequency cannot be described.

Conclusions

1. Zero-dimensional models can capture growing oscillations similar to the breathing mode.
2. The current model may be incomplete: no useful criteria for instability are presented, and the real frequency cannot be predicted.
3. 1D effects may need to be introduced to produce conditional instability.

Acknowledgments

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