The Near-Anode Region in a Hybrid, Direct-Kinetic Hall Thruster Simulation

Astrid L. Raisanen and Iain D. Boyd
The University Of Michigan, Department of Aerospace Engineering, Ann Arbor, MI

Hall Thruster Background Information

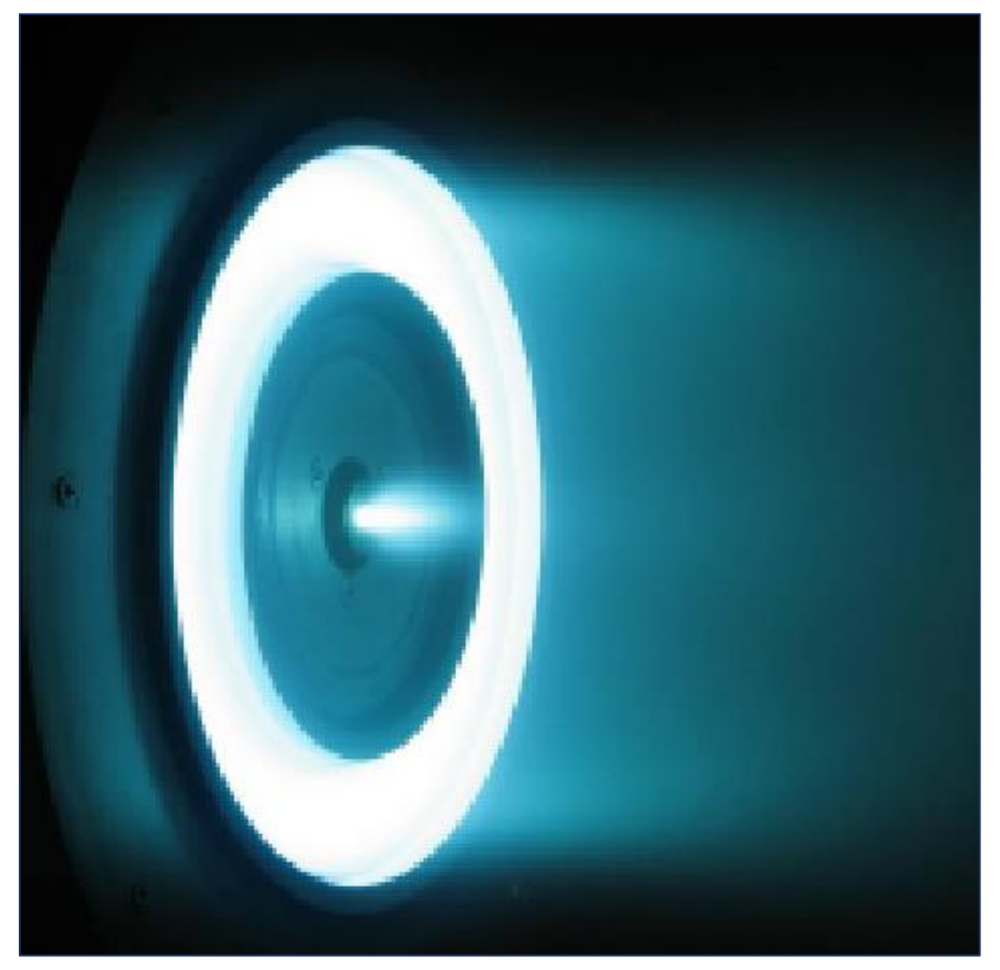


Fig. 1: Xenon Hall thruster [1]

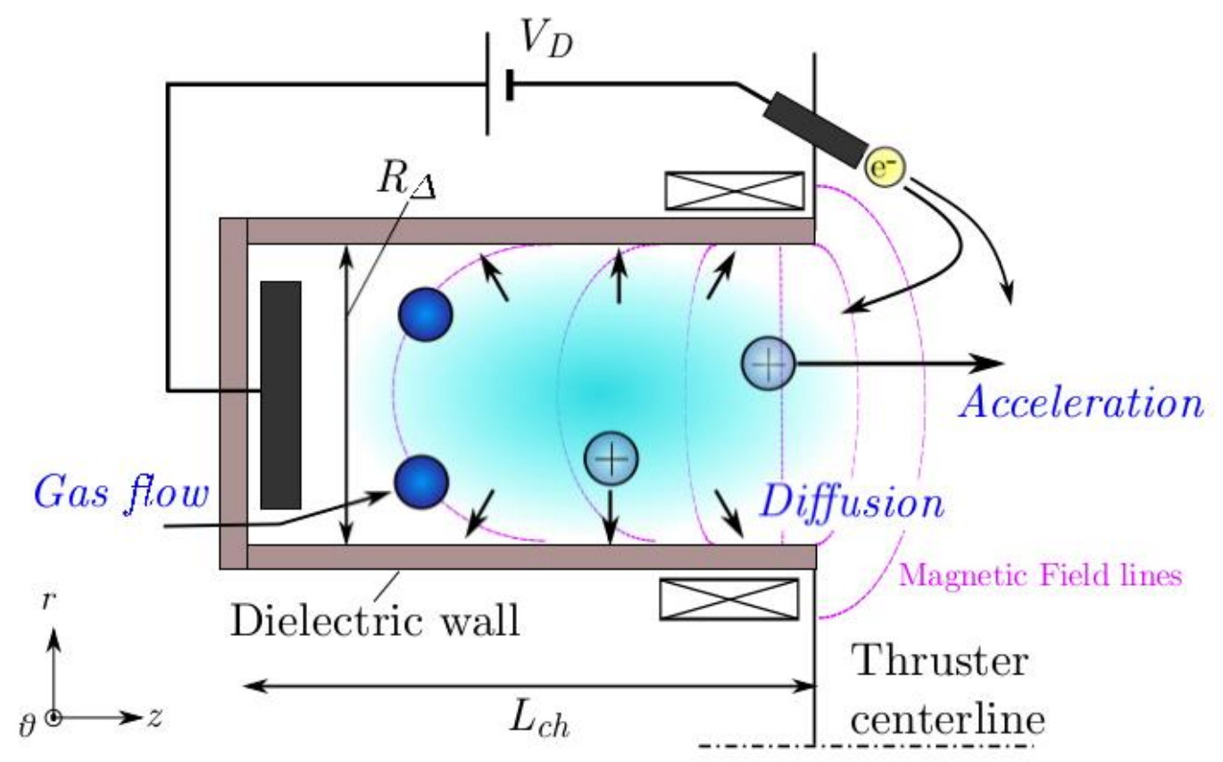


Fig. 2: Hall thruster channel [2]

Hall thrusters generate thrust in the following manner:

- A static, primarily radial magnetic field is applied between two electromagnets.
- Neutral gas (typically xenon) is injected at the anode side of the thruster.
- An external cathode ejects electrons into the flow field.
- Due to the potential drop across the thruster, electrons travel toward the anode. However, they become magnetized and are therefore impeded by the magnetic field.
- Electrons collide with neutral atoms, ionizing them. Ions have a large gyroradius and therefore remain largely un-magnetized.
- Ions are accelerated out of the thruster via the electric field, generating thrust.

Introduction: Hybrid- Direct Kinetic Simulation

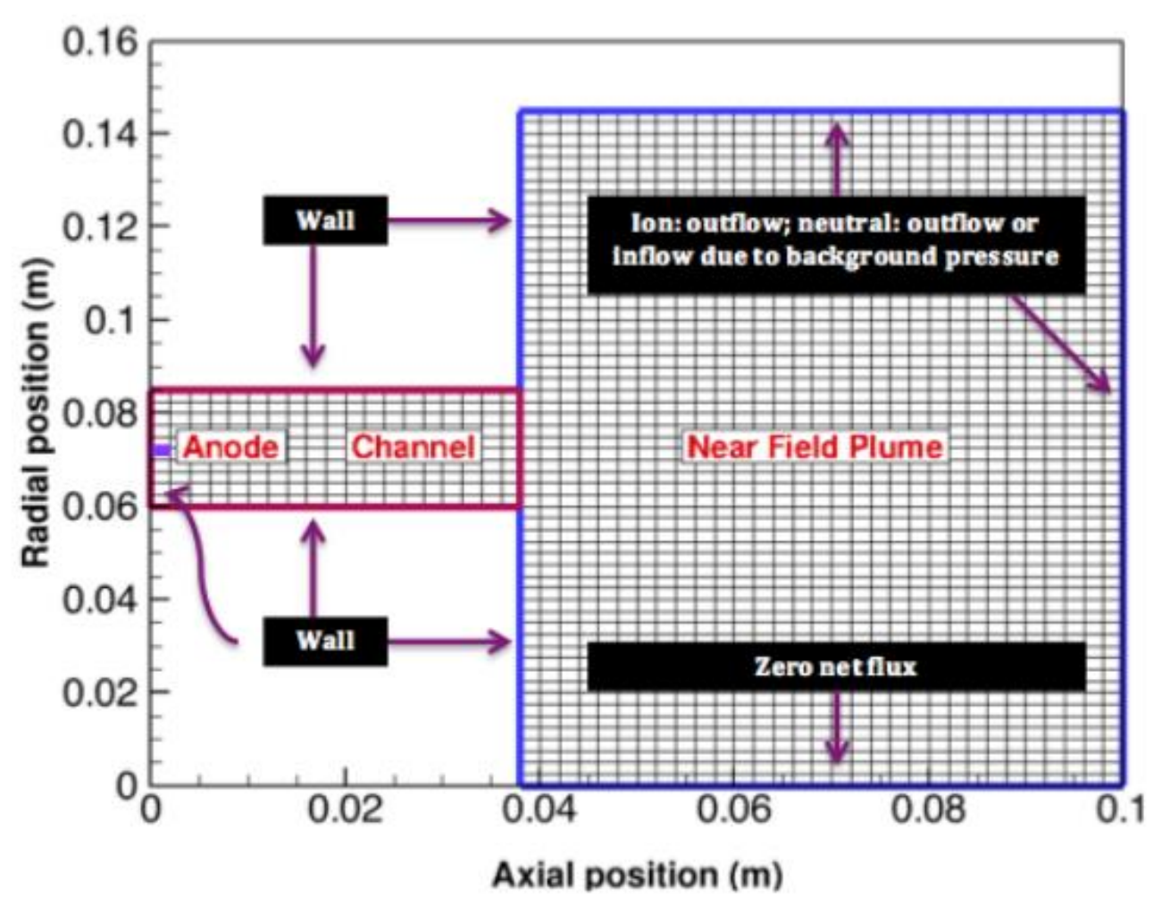


Figure 3: Direct Kinetic simulation domain

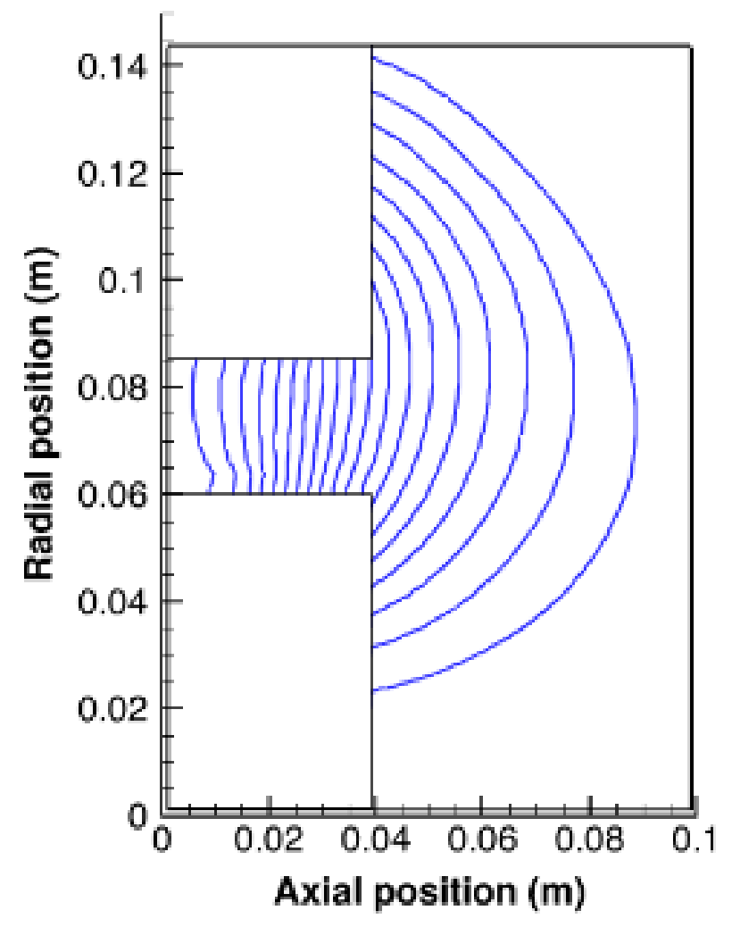


Figure 4: Electron simulation domain

A hybrid-Direct Kinetic (DK) simulation is under development at the University of Michigan. A two-dimensional DK solver (Figure 3) is coupled with a quasi-one dimensional electron solver (Figure 4).

Ion and neutral Boltzmann equations are described by:

$$\frac{\partial f_i}{\partial t} + v_z\frac{\partial f_i}{\partial z} + v_r\frac{\partial f_i}{\partial r} + \frac{eE_z}{m_i}\frac{\partial f_i}{\partial z} + \frac{eE_r}{m_i}\frac{\partial f_i}{\partial r} = S_i \qquad \frac{\partial f_n}{\partial t} + v_z\frac{\partial f_n}{\partial z} + v_r\frac{\partial f_n}{\partial r} = S_n$$

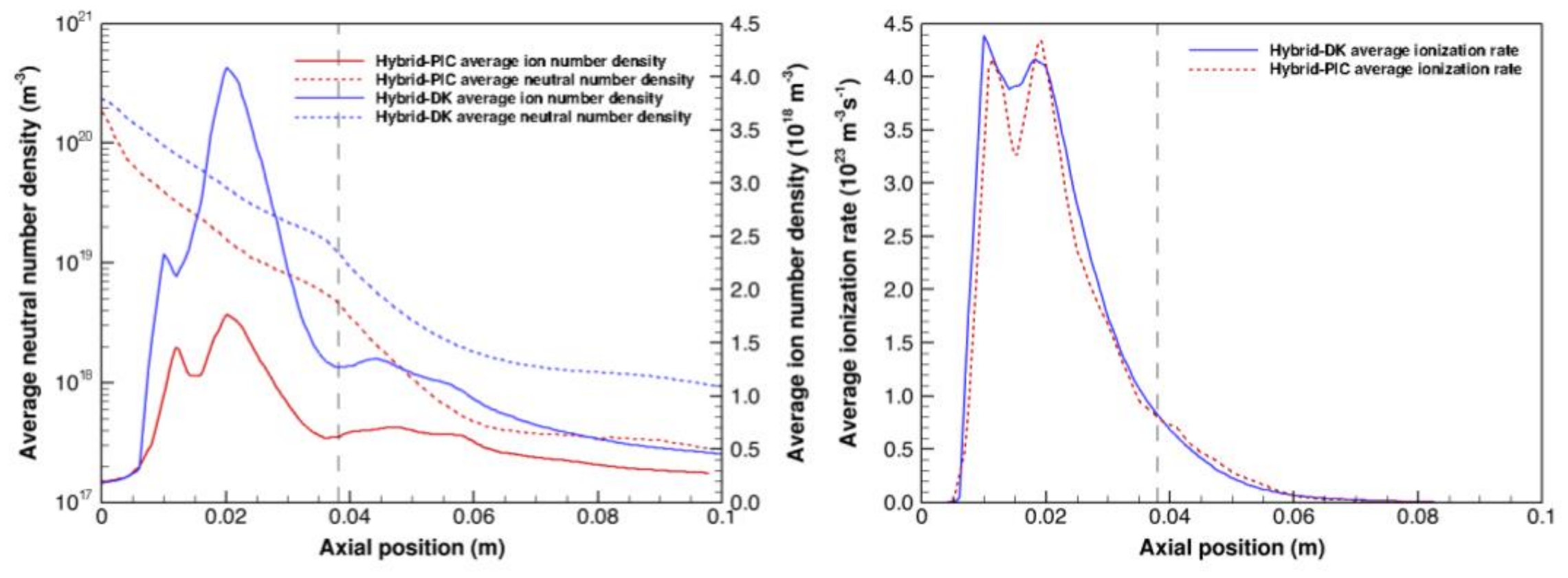
Integration of the momentum equation along a magnetic field line results in the thermalized potential, which is a reduced description of the plasma potential:

$$\phi^*(\lambda) = \phi - \frac{k_b T_e}{e}\ln\frac{n_e}{n_e^*}$$

Modeling and Theory

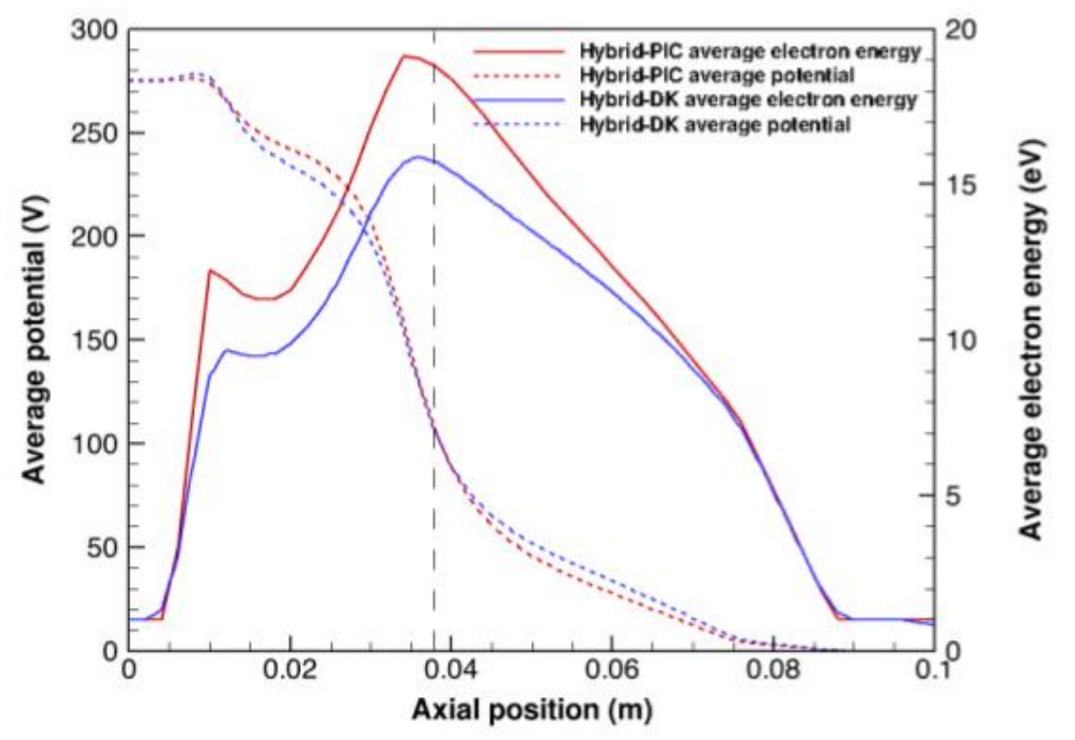
Benchmarking

The hybrid-DK simulation was benchmarked with Koo and Boyd's hybrid-Particle-in-Cell (PIC) simulation [3], [4]. Results in Figure 5 indicate that the DK simulation performs adequately in comparison to the PIC simulation.



(a) Number density.

(b) Ionization rate.



(c) Potential and electron mean energy.

Figure 5: Axial profiles of average plasma properties along the channel centerline at t = 0.94 ms [4]

However, anode coupling (Figure 6) may contribute to the damped hybrid-DK discharge current shown in Figure 7.

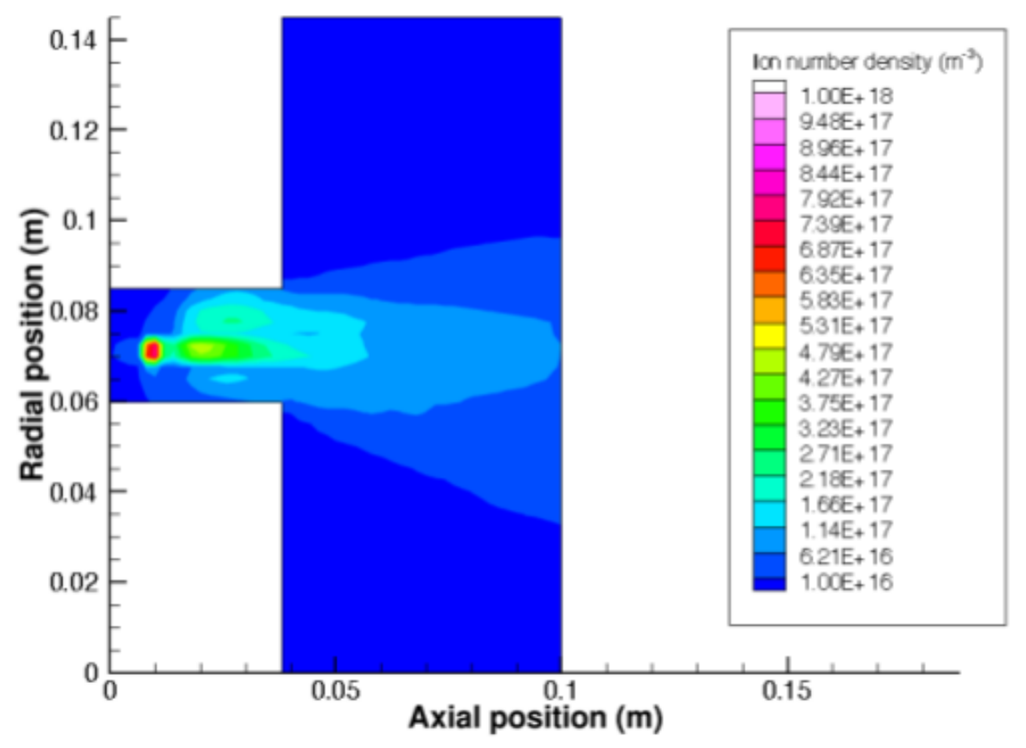


Figure 6: DK instantaneous ion number density at t = 1.0 ms [4]

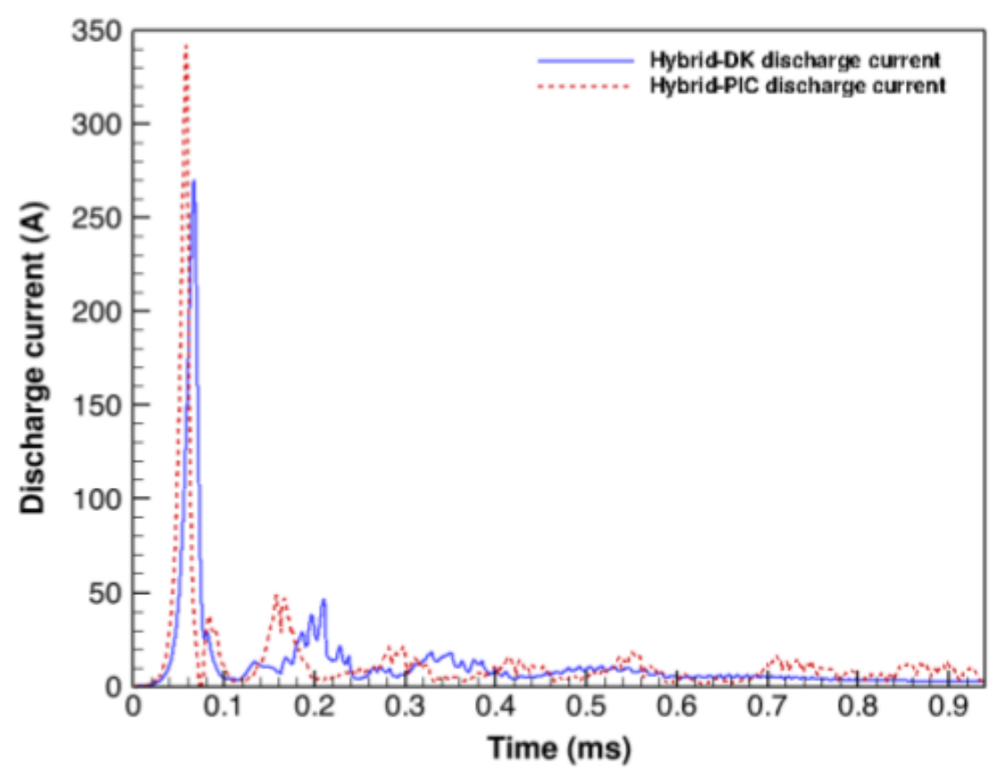


Figure 7: Hybrid-PIC and hybrid-DK discharge current versus time [4]

Near-anode Region

To address coupling at the anode λ-line, a study is conducted on the electron boundary conditions in the near-anode region. Application of a Neumann electron temperature boundary condition in conjunction with a sheath condition at the anode λ-line offers improvement in the potential profile, but it does not result in a satisfactory solution. The calculation for the sheath potential is identical to that in HPHall [5]:

$$\phi_a = \frac{-2}{3}\epsilon\ln\left[\frac{I_a}{A_e n_e\sqrt{\frac{e\epsilon}{3\pi m_e}}} + e^{-0.5}\sqrt{\frac{2\pi m_e}{m_i}}\right]$$

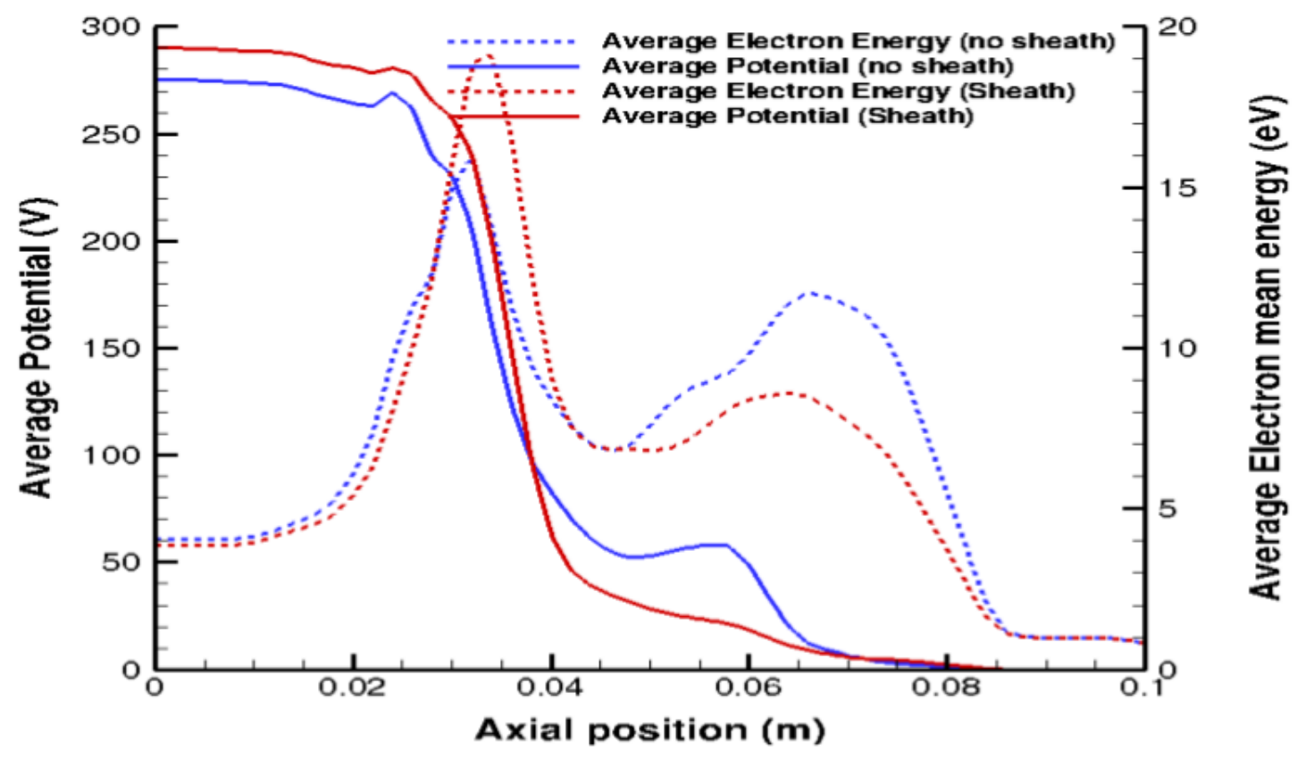


Figure 8: Average axial profiles of potential and electron energy at the channel centerline for cases with and without an anode sheath condition at t = 1.0 ms

The following sheath condition, applied at the simulation boundary where the anode surface is located (z = 0), may provide closure [6]. The key is to ensure that properties calculated in the anode λ-cell are consistent.

- Apply a Dirichlet boundary condition for the potential at the wall: $\phi_{wall} = V_d$
- Assuming that the sheath is thin, ensure that the current is balanced at the anode and sheath edge. Calculate the sheath potential:

$$n_{e,sh}u_{ze,sh} = -n_{e,sh}\exp\left(\frac{-e\phi_{sh}}{T_{e,sh}}\right)\sqrt{\frac{T_{e,sh}}{2\pi m_e}}$$

- Calculate the electron energy flux deposited at the anode to determine the electron temperature:

$$q_{ze,sh} = n_{e,sh}u_{ze,sh}(2T_{e,sh} + e\phi_{sh}) \qquad q_{ze,anode} = n_{e,sh}u_{ze,sh}(2T_{e,sh})$$

Conclusion and Future Work

In recent published work, a hybrid-DK simulation was benchmarked with a hybrid-PIC simulation, both of which utilized identical electron algorithms [3]. However, it became apparent that the physics in the near-anode region of the hybrid-DK domain must be better resolved to avoid anode coupling.

Preliminary results indicate that an electron-repelling sheath at the anode should be accounted for in the electron model. An approximate sheath boundary condition, similar to that applied in the hybrid-PIC simulation HPHall, is installed but is determined insufficient for the hybrid-DK simulation since it does not account for the electron energy flux in the near-anode region.

In the near future, a more accurate sheath boundary condition will be installed, ensuring current continuity and accounting for the energy flux at the thruster anode.

References

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Acknowledgments

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