Probabilistic evaluation of closure models for the Hall thruster anomalous collision frequency

**Problem:** Current Hall thruster simulations are not predictive due to incomplete understanding of electron transport physics.

**Hall thrusters**
Annular $\vec{E} \times \vec{B}$ discharge used to accelerate ions for spacecraft propulsion

**Anomalous electron transport**
Electrons diffuse across magnetic field lines much faster than classical theory predicts, so simulations cannot match experiment without hand-tuning. Model as extra “anomalous collision frequency” ($\nu_{AN}$) in electron momentum equation

\[ \nu_{AN} = \frac{1}{K} \cdot \text{coefficients: } \{K\} \]

**Ohm's law:**
\[ (\nu_e + \nu_{AN})^\frac{\mu_e}{q} = qn_e\vec{E} + \nabla P_e - \vec{J}_e \times \vec{B} \]

**Question:** How can we develop and test models of anomalous transport while accounting for model uncertainty?

**Approach:** Use validated simulations as surrogate data to calibrate and test models of anomalous transport

**Application:** Evaluate multiple algebraic models to determine if any are predictive and extensible

**Discussion:** Examine reasons for low performance

**Axial properties for data-driven model**

**Conclusions:**
- Algebraic models tend to under-predict performance when calibrated on steady-state data
- Breathing-like oscillations consistently reproduced
- Bayesian techniques can quantify model uncertainty

**Nomenclature:**
- $\nu_{AN}$: Anomalous collision freq.
- $m_e$: Electron mass
- $n_e$: Electron number density
- $P_e$: Electron pressure
- $\nu_e$: Electron drift speed
- $\nu_{AN}$: Anomalous collision freq.
- $\omega_c$: Electron cyclotron freq.
- $\beta$: Magnetic field
- $E$: Electric field
- $\vec{J}_e$: Electron current density
- $T_e$: Electron temp.
- $\nu_c$: Classical electron collision freq.
- $u_i$: Ion velocity
- $c_a$: Ion acoustic speed

**References:**